G5 高等準備金，價值評估與企業風險管理

1.(5 分)

根據以下資訊回答下列問題

	累積已付賠款 ($ 000)		
意外年度'	12	24	36
2012	2,750	4,250	5,100
2013	2,700	4,300	
2014	2,900		

意外年度預期之損失發展(growth function)為 Weibull function 之形式:

$G(x|\omega,\theta) = 1 - \exp(-(x/\theta)^{\omega})$ ， ω=1.5， θ=20

(1)(2 分)

請採用 LDF 法計算意外年度 2012 年到 2014 年賠款準備金之 Process Standard Deviation

(2)(3 分)

請針對損失發展年度畫出 normalized residuals，並根據此圖形說明採用 Weibull model 之適當性

【參考解答】

(1)

Process st dev = $\sqrt{R\sigma^2}$ $\sigma^2 = 1/(n-p)\sum(c-\mu)^2/\mu$ n=6 p=2+3=5

↑ ↑

G,W Ult 2010, Ult 2011, Ult 2012

G(6)=1-$e^{(-(6/20)^{\wedge} 1.5)}$ = .1515

G(18) = .5742

G(30) = .8407

* Check if truncation is needed:

- Extend Δ out 3 years G(66) = .9975
- G(66) is reasonably close to 1 and so the function does not need to be truncated

AY	G(x)	LDF = 1/G(x)	Ult = Cum Pd×LDF	Reserve
2010	0.8407	1.1895	6,066.4	966.4
2011	0.5742	1.7416	7,488.7	3,188.7
2012	0.1515	6.6007	19,142	16,242
				20,397.1

avg age:	(6)	(18)	(30)
expected	12	24	36
10	919	2564.3	1616.7
11	1134.5	3165.5	1995.7
12	2900	8091.3	5101.3

$\mu = [G(y) - G(x)]$ Ult AY

1995.7 = (.8407-.5742)(7488.7)

actual

10	2750	1500	850
11	2700	1600	
12	2900		↖4300-2700

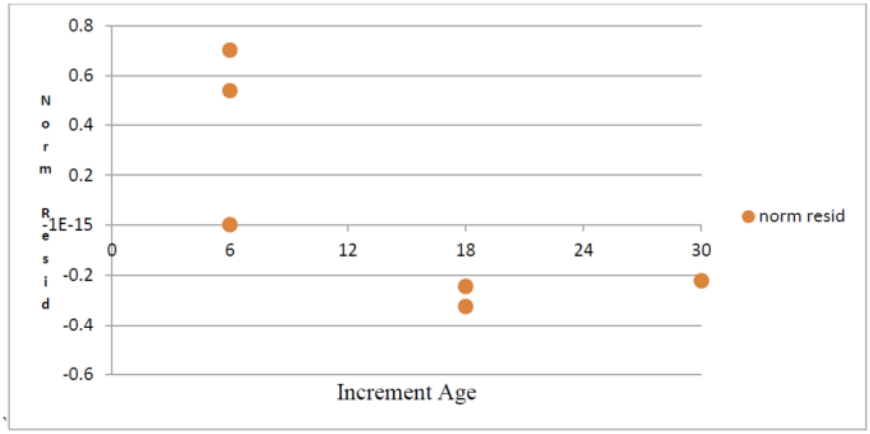
$\sigma^2 = 1/(6-5) \times \left[\frac{(2750-919)^2}{919} + \frac{(2700-1134.5)^2}{1134.5} + \ldots\right] = 7387.84$

process standard dev $= \sqrt{R\sigma^2} = \sqrt{7387.84 * 20397.1} = 12{,}275.607$

(2)

$\varepsilon = (c - \mu)/\sqrt{\mu\sigma\text{^}2}$

6	18	30
0.7027	-0.245	-0.222
0.5407	-0.324	↑
		(850-
0		1616.7)/(7387.4×1616.7)^0.5



若 Weibull function 是適當的，我們可以看到 residuals 應該接近 0，但在圖形中看到了隨者損失發展年齡增加，residual 由正數變為負數，因此 Weibull function 不不適當。

2.(3 分)

請根據 Chain Ladder 方法回答下列問題：

(1)請敘述由 Mack 所描述之 Chain Ladder 方法下的三個假設(1.5 分)

(2)請根據下列情境討論是否有違反 Mack 的任何假設(1.5 分)

a. 第一個已付損失法展三角形顯示出因採用新的理賠管理軟體導致較快的理賠結案趨勢

b. 根據第二個已付損失法展三角形，精算師發現採用所有年度的 volume-weighted average 是選取損失發展因子(Loss development factor)最適當之方法

c. 第三個已付損失法展三角形顯示 36-48 個月的損失法展因子與 24-36 個月的損失法展因子呈現反比關係，此相關性並非隨機。

【參考解答】

(1)

a. 下一段發展期間之預期損失發展增量等於累積至此發展期間之損失乘以損失法展因子

b. 任一意外年度之損失與其他意外年度之損失為獨立關係

c. 下一段發展期間之預期損失 variance 與累積至此發展期間之損失 variance 呈現正比關係

(2)

a. 違反以上的第二個假設，因為新的理賠管理軟體會影響所有意外年度之損失，因此所有意外年度之損失並不具獨立性。

b. 不違反任何假設。

c. 違反以上第一個假設，下一段發展期間之預期損失發展增量只與累積至此發展期間之損失有相關，但損失發展因子應具獨立性。

3.(5 分)

請解釋與說明五個技術上的困難，造成再保險公司準備金的預估比簽單保險公司困難。

【參考解答】

a. 再保險公司收到報案的時間比簽單公司更久 (longer reporting lag)

b. 資料的同質性過低，無法用傳統的精算方法計算

c. 產業資料不實用

d. 再保險公司所收到的賠案資料不充分(例如無法區分意外年度)

e. 數據編碼(IT coding)與 IT 系統無法滿足再保險公司較複雜的需求

4.(5 分)

根據以下至 12/31/2014 為止的兩個比例型再保合約之資料：

合約	起保日	合約承保基礎	簽單保費	已發生賠款
No. 1	1/1/2012	損失發生時間 (loss occurring)	80,000	50,000
No. 2	7/1/2012	簽單時間 (policies written)	100,000	20,000

意外發展年齡 (月份)	最終損失發展因子
36	1.25
24	2.00
12	3.00

- 兩個比例型再保合約皆為一年期
- 再保險合約下之簽單保險合約(underlying policies)皆為一年期之事故發生制合約，合約之生效日平均分布在一年當中
- 再保合約 No.1 之佣金、仲介費與內部費用為保費之 5%
- 再保合約 No.2 之佣金、仲介費與內部費用為保費之 20%
- 損失發展因子可適用在該意外外年度的所有暴險(exposure)

(1)請採用 Standard Buhlmann 方法計算兩個再保合約結合後之最終賠款(2 分)

(2)具有長尾風險(long-tail exposure)特性時，若要計算 IBNR，何種情況下是 chian-ladder 方法勝過 Standard-Buhlmann 方法(1 分)

(3)具有長尾風險(long-tail exposure)特性時，若要計算 IBNR，何種情況下是 Standard-Buhlmann 方法勝過 chain-ladder 方法(1 分)

(4)請說明 Standard-Buhlmann 法相較於 Bornhuetter-Ferguson 法的主要創新(1 分)

【參考解答】

(1) 移除費用

Treaty 1 = 80000 * (1-5%) = 76000

Treaty 2 = 100000 * (1-20%) = 80000

在 12/31/2014, Treaty 1 是發展至 36 個月，Treaty 2 是發展至 24 個月(因平均簽單日是 1/1/2013)

Use-up premium = 76000 * (1/1.25) + 80000 * (1/2) = 100800

所有已發生賠款 = 50000 + 20000 = 70000

ELR = 70000/108000 = 0.694

IBNR = 0.694 * (76000 * (1-1/1.25) + 80000 * (1-1/2)) = 38308.8

最終賠款=IBNR+已發生賠款=38308.8+70000 = 108308.8

(2)將所有年度的保費調整到同樣的費率水準(rate level)

(3)當已發生賠款變動很大，但 ELR(預期損失率調整至同樣費率水準)是較穩定的

(4)ELR(預期損失率)是根據過去的損失經驗，而非武斷或經判斷選定的

5.(3 分)

保險合約之自負額(deductible)為 500,000，根據以下資料進行計算：

	損失發展因子			
	12-24	24-36	36-48	48-60
Unlimited	1.912	1.316	X	1.077
Limited to 500,000	1.900	1.300	1.125	1.050

Limited Severity Relativities R^L				
12	24	36	48	60
0.980	0.974	0.962	0.944	0.920

(1)請計算上表之 X 值(1.5 分)

(2)請計算超過自付額 500,000(excess of 500,000)之損失於 36-48 個月之損失發展因子(1.5 分)

【參考解答】

(1) 1.125 = X * (0.944/0.962)

X = 1.1465

(2) 1.1465 = 1.125 * 0.962 + LDF * (1-0.962)

LDF = 1.69

6.(5 分)

藉由根據增量損失(incremental loss)建立 GLM(generalized linear model)，下方之損失發展三角形將用於 bootstrap process，請回答下列問題：

	累積已發生賠款		
意外年度	12	24	36
2012	300,000	350,000	400,000
2013	400,000	450,000	
2014	550,000		

(1)請建立 log-link 損失發展三角形，以用於計算 GLM 中之參數(2 分)

(2)假設意外年度 2012 年於發展至 24 個月之累積已發生賠款為 200,000，而非 350,000，損失發展三角形中之其他值均不變，請解釋若要採用修改後之損失發展三角形建立 GLM bootstrap model 之困難(2 分)

(3)請根據(2)修改後之損失發展三角形建立 log-link 損失發展三角形，以用於計算 GLM 中之參數；並請簡述如何調整 GLM 的適配值(fitted values)(1 分)

【參考解答】

(1)

	增量賠款		
意外年度	12	24	36
2012	300,000	50,000	50,000
2013	400,000	50,000	
2014	550,000		
log-link triangle = ln (增量賠款)			
意外年度	12	24	36
2012	12.61	10.82	10.82
2013	12.90	10.82	
2014	13.22		

(2)

	增量賠款		
意外年度	12	24	36
2012	300,000	- 100,000	200,000
2013	400,000	50,000	
2014	550,000		

GLM bootstrap model 假設所有增量損失均不為負數

(3)

將(2)表格中所有值均加上 100,000

	增量賠款		
意外年度	12	24	36
2012	400,000	0	300,000
2013	500,000	150,000	
2014	650,000		
log-link triangle = ln (增量賠款)			
	增量賠款		
意外年度	12	24	36
2012	12.90	-	12.61
2013	13.12	11.92	
2014	13.38		

7.(3 分)

請根據以下保險公司之財務資訊，依 dividend discount model 計算公司於 12/31/2014 之價值

		預估值				
	2014	2015	2016	2017	2018	2019
淨損益 (Net Income)	200	200	220	235	240	250
期初股東權益 (Beginning Equity)	2,110	2,230	2,350	2,482	2,623	2,767
期末股東權益 (Ending Equity)	2,230	2,350	2,482	2,623	2,767	2,917

- 預估市場上同性質之保險公司之股東權益報酬率(ROE) = 10%
- 無風險利率(risk-free rate) = 2%
- 同性質保險公司之 beta (β) = 0.85
- 公司計畫之再資比率為 plowback ratio = 60%
- 採用 CAPM 決定風險調整後之折現率(risk-adjusted discount rate)

【參考解答】

Discount rate = 0.02+0.85*(0.1-0.02) = 0.088

dividend = 1-60% = 40%

	2015	2016	2017	2018	2019
Dividend	80	88	94	96	100
ROE = NI/Beginning Equity	0.09	0.09	0.09	0.09	0.09

選擇 ROE=0.09

g = p * ROE = 60% * 0.09 = 0.054

公司價值 = $\frac{80}{1.088}+\frac{88}{1.088^2}+\frac{94}{1.088^3}+\frac{96}{1.088^4}+\frac{100}{1.088^5}+\frac{100*(1+0.054)}{1.088^5(0.088-0.054)}$ = 2388.33

8.(3 分)

請根據以下追溯費率之保險合約回答下列問題：

	For Losses Capped at Per Accident Limit		
Retro Adjustment Period	Insurance Charge at Retro Maximum	Insurance Charge at Retro Minimum	Loss Elimination Ratio from Per Accident Limit
1	0.112	0.005	4.20%
2	0.140	0.004	5.20%
3	0.152	0.004	6.00%
4 ~	0.155	0.003	6.40%

(1)請針對每個追溯調整期間(Retro Adjustment Period)計算 capped loss ratio(2 分)

(2)請解釋上題 capped loss ratio 所呈現之趨勢(1 分)

【參考解答】

(1)

	For Losses Capped at Per Accident Limit				
Retro Adjustment Period	Insurance Charge at Retro Maximum (1)	Insurance Charge at Retro Minimum (2)	Loss Elimination Ratio from Per Accident Limit (3)	Eliminated by Retro Max&Min (4) = (1) - (2)	Capped loss ratio (5) = 1 - (3) - (4)
1	0.112	0.005	4.20%	0.107	0.851
2	0.140	0.004	5.20%	0.136	0.812
3	0.152	0.004	6.00%	0.148	0.792
4 ~	0.155	0.003	6.40%	0.152	0.784

(2) capped loss ratio 呈現逐漸下降之趨勢，因為隨者損失發展期間逐漸成熟，將有逾大戰比的損失是在限額之外。

9.(2 分)

請根據以下保險公司發行之債券回答下列問題：

	至到期年度	評等	信用風險資本需求($ 000,000)	發行債券保險公司之違約率
債券A	1	BBB	60	0.30%
債券B	3	BBB	160	0.30%

假設在最佳風險分散(diversification)的狀況下，預估相關係數(correlation coefficients) = 0.5

(1)請說明與債券相關信用風險之三個主要來源(1 分)

(2)假設以上兩個債券由同一保險公司發行之狀況下，計算因信用風險所需之資本(1 分)

【參考解答】

(1)

違約風險(default risk)

調降信用評等之風險(downgrade risk)

集中風險(concentration risk)

(2) 債券由同一保險公司發行，因此相關係數等於 1，資本需求=60+160=220

10.(3 分)

為了維持保險公司 A+的信用評等，保險公司須符合最低資本要求之規範，最低資本要求為超過 5%年初之 US GAAP 股東權益(US GAAP Equity)：

	2015	2016	2017
期初 GAAP Equity	5,000,000		
淨損益 (Net Income)	380,000	450,000	520,000

(1)請計算 2017 年的 Free Cash Flow to Equity (FCFE) (2 分)

(2)請解釋為何 FCFE 評價方法不受到保險公司於 2015 年賠款準備金上升之影響(1 分)

【參考解答】

(1)

		2015	2016	2017
期初 GAAP Equity	(1)	5,000,000	5,250,000	5,512,500
資本需求	(2)	5,250,000	5,512,500	5,788,125
淨損益 (Net Income)	(3)	380,000	450,000	520,000
FCFE = (3) - ((2) - (1))		130,000	187,500	**244,375**

(2)賠款準備金上升將同時影響淨損益與資本需求，在計算 FCFE 時互相抵銷因此 FCFE 評價方法不受影響。

11.(2 分，以下各佔 0.5 分)

請依下列問題回答：

(1)請說明市場風險(market risk)與信用風險(credit risk)

(2)請描述三種保險公司可能面對之市場風險

(3)請分別說明市場風險與信用風險如何影響投資資產(invested asset)與保險負債(insurance liability)

(4)假設以下各風險之相關性為非負(non-negative)，請根據下列資料計算總風險之範圍

資產之市場風險 = 150

資產之信用風險 = 80

負債之市場風險 = 100

負債之信用風險 = 50

【參考解答】

(1)市場風險：財務變數的改變導致資產或負債價值的變動

信用風險：交易對手被調降信評或違約之風險

(2)Concentration risk(過度集中風險)、Interest rate risk(利率風險)、Reinvestment risk(再投資風險)、Asset-Liability mismatch risk(資產負債不對稱風險)。

(3)市場風險會影響股票與債券之價值，同時也影響負債面之通貨膨脹率

信用風險影響債券之信用評等與違約率，同時也影響再保人之違約率

(4) Min risk = $\sqrt{150^2+80^2+100^2+50^2}$ = 203.47

Max risk = 150+80+100+50 =380

12.(3 分)

請依下方已發生賠款發展模式下列問題回答：

意外發展年度(月份)			
12	24	36	48
20%	50%	70%	85%

- 損失發展模式乃依據過度分散之卜瓦松模型(over-dispersed Poisson Model)，分散參數 ϕ=1.3
- 意外年度 2014 年預期最終損失 = 1,800,000

(1)請針對意外年度 2014 年計算增量已發生賠款(incremental reported loss)於 36-48 個月的變異數(variance)(2 分)

(2)請解釋為何於過度分散之卜瓦松模型(over-dispersed Poisson Model)，使用已付賠款或理賠件數相對於使用已發生賠款較為適當(1 分)

【參考解答】

(1)預期 36-48 個月發展之賠款金額=1,800,000 * (85%-70%) = 270,000

Variance = 270,000 * 1.3 = 351,000

(2)已付賠款與理賠件數通常不會有負向的損失發展，因為不受已報未付賠款調整之影響，因過度分散之卜瓦松模型受限於各欄位中之增量損失加總須為正數

13.(2 分)

目前保險公司計畫升級理賠系統，此專案預計於數個月內完成，同時這段期間將於某些時段定期關閉理賠系統，此系統之設計與安裝均委託外部系統商完成

(1)請說營運風險(operational risk)(1 分)

(2)請描述此專案可能面臨之兩個營運風險，同時提出方法以減少這兩個營運風險(1 分)

【參考解答】

(1)營運風險包含來自人為、系統或流程所引起之風險，包含法律風險，但不包含商譽或策略

風險

(2)營運中斷或系統中斷之風險：應有一套備用系統以供正常營運，將建置中與正常營運系統完全劃分

外部詐騙之風險：因為此系統乃委託外部廠商完成，外部廠商有竊取公司資料的機會，應設定密碼與資料權限以防範此風險

14.(2 分)

假設 A 保險公司有以下資訊:

已發生損失率(Incurred Loss Ratio)

Accident Year	12 Months	24 Months
2012	0.225	0.445
2013	0.425	0.655
2014	0.413	0.651
2015	0.255	

(1)使用 Least Squares Method 估計 AY 2015 年於 24 個月之損失率。(1 分)

(2)假設 A 保險公司採用 Link Ratio Method 及 Budgeted loss Ratio Method 的平均數估計 AY2015 年於 24 個月之損失率,請依據第(1)小題計算結果評估此方法是否為最佳(optimal)方法。(1 分)

【參考解答】

(1)

$\bar{x}$=0.354, $\bar{y}$=0.584, $\overline{x^2}$=0.134, $\overline{xy}$=0.216

$\hat{b}$=(0.216-0.354*0.584)/(0.134-0.354^2)=1.067

$\hat{a} = \bar{y} - \hat{b}\bar{x}$ =0.206

$\hat{y}$=0.206+1.067*0.255=0.478

(2)

最佳方法即為 Least Squares method. 而 LS method 係為 Link Ratio Method 及 Budgeted loss Ratio Method 之加權，權重分別為 b/c 及(1-b/c)。

依據第 1 小題計算結果:b=1.067,c=0.548/0.354=1.65,

Link Ratio Method 權重=1.067/1.65=0.647(不等於 0.5)
因此採用 Link Ratio Method 及 Budgeted loss Ratio Method 的平均數估計不是最佳方法。

15.(5 分)
假設 A 保險公司有以下資訊:

Incremental Paid Losses

Accident Year	12 Months	24 Months	36 Months	48 Months	60 Months	72 Months
2010	1000	800	600	300	200	100
2011	1200	600	500	400	300	
2012	1000	1500	500	300		
2013	1300	1000	600			
2014	1500	1200				
2015	1400					

Accident Year	Earned Premium
2010	3000
2011	4000
2012	5000
2013	6000
2014	7000
2015	8000

請計算AY 2015年之Hurlimann’s Optimal Credibility Reserve。

【參考解答】

m_1=22.42%=(1000+1200+1000+1300+1500+1400)/(3000+4000+5000+6000+7000+8000)
m_2=20.40% , m_3=12.22% , m_4=8.33% , m_5=7.14% , m_6=3.33%
ELR=22.42%+20.40%+12.22%+8.33%+7.14%+3.33%=73.85%
p=22.42%/73.85%=0.304
q=1-p=0.696
z=0.304/(0.304+0.304^0.5)=0.355
R^{ind}=0.696*1400/0.304=3205
R^{coll}=0.696*(8000*0.7385)=4112

R^C=0.355*3205+(1-0.355*4112)=3790

16.(2 分)
請說明採用Over-Dispersed Poisson Bootstrap Model時常見的4種資料問題，並針對每一個問題提供一個解決方式。

【參考解答】

Sample Answer 1
1.Negative incremental value: limit incremental losses to zero
2.Missing values/incomplete data: estimate missing values using surrounding values
3.Heteroscedasticity: stratified sampling is accomplished by organizing the development periods by group with homogeneous variance within each group and them sampling with replacement only from the residuals in each group
4.Exposures that have changed dramatically over the years: modify data to get pure premiums and multiply the residuals by the exposures by year after the process variance step

Sample Answer 2
1.Non-zero sum of residuals: add a single constant to all residuals so that the sum of the adjusted residuals is zero
2.Outliers/extreme values: exclude outliers from the average age-to-age factors and residual calculations, but re-sample the corresponding incremental when simulating triangles
3.Heteroecthesious data/misshapen data/partial year/interim evaluation dates: project future incremental values before applying model; need to annualize then de-annualize results
4.Lack of residuals to sample from/lack of extreme residuals: need to parameterize a distribution from which to sample from

17.(5 分)
下列損失發展三角形為損失限額300,000元之資料，請評估損失限額150,000元之12-Ult及24-Ult損失發展因子

	Cum Incurred Losses(000)		
Accident Year	12 Months	24 Months	36 Months

2013	20,000	45,000	52,000
2014	22,000	46,000	
2015	24,000		

相關假設條件如下:

AY 年度趨勢值=10%，CY年度趨勢值=0%，損失金額假設服從指數分配且AY2015年12、24及36發展月數平均損失金額分別為100,000、150,000及200,000元。

【參考解答】

	AY TREND
2013	1.000
2014	1.100
2015	1.210

CY TREND	12 Months	24 Months	36 Months
2013	1.000	1.000	1.000
2014	1.000	1.000	1.000
2015	1.000	1.000	1.000

TREND	12 Months	24 Months	36 Months
2013	1.000	1.000	1.000
2014	1.100	1.100	1.100
2015	1.210	1.210	1.210

Unlimit Mean	12 Months	24 Months	36 Months
2013	82,645	123,967	165,289
2014	90,909	136,364	181,818
2015	100,000	150,000	200,000

Eg. for AY 2014, age 12 Months: 90909=100000*1.1/1.21

L=300,000,B=150,000

LEV (L)	12 Months	24 Months	36 Months
2013	80,453	112,944	138,374
2014	87,556	121,254	
2015	95,021		

Eg. for AY 2014, age 12 Months: 87556=90909*[1-(e^(-300000/90909))]

LEV (B)	12 Months	24 Months	36 Months
2013			
2014			
2015	77,687	94,818	105,527

Eg. for AY 2015, age 12 Months: 77687=100000*[1-(e^(-150000/100000))]

Finally, we can calculate C'

C'	12 Months	24 Months	36 Months
2013	19,312	37,778	39,656
2014	19,520	35,971	
2015	19,622		

Eg. for AY 2014, age 12 Months: 19520=22000*77687/87556

Now, we calculate the LDFs:

LDF_{12} =(37778+35971)/(19312+19520)=1.899

LDF_{24} =39656/37778=1.050

LDFs to Ultimate

LDF_{12}=1.899*1.050=2.066

LDF_{24} =1.050

18.(2 分)

(1) 請說明 Mack Method 的 3 個假設。(1 分)

(2) 請就下列情況評論是否符合 Mack Method 的假設，若不符合請明確指出違反之假設：

(a)精算人員於分析損失發展三角形時發現損失發展因子選定的最佳方式是採用算術平均方式估計。(0.5 分)

(b)精算人員於分析損失發展三角形時發現，當某意外年度之 12-24 損失發展因子高於 12-24 損失發展因子平均數時，該意外年度之 24-36 損失發展因子會低於 24-36 損失發展因子平均數。(0.5 分)

【參考解答】

(1)

i. Expected incremental losses are proportional to losses reported to date

ii. Variance of incremental losses are proportional to losses reported to date

iii. Losses in one accident year are independent of losses in another accident year

(2)

a. 不符合。違反上述第二個假設，若符合第二個假設則損失發展因子選定的最佳方式應為賠款金額加權平均方式估計。

b. 不符合。違反上述第一個假設， 因為本期損失金額與前期發展因子相關而非僅與 losses reported to date 相關。

19.(3 分)

請依據 Mack 之 Measuring the Variability of Chain Ladder Reserve Estimates 回答問題。假設 A 保險公司損失三角形資料如下，該公司精算人員發現已付賠款增量之變異數與已付賠款增量平方呈現比例關係，請說明採用之最佳損失發展因子並計算該公司於 2015 年底之賠款準備金金額。

Cumulative Paid Losses

Accident Year	12 Months	24 Months	36 Months	48 Months
2012	1000	1150	1300	1400
2013	1000	1200	1300	
2014	1000	1100		
2015	1000			

【參考解答】

因為已付賠款增量之變異數與已付賠款增量平方呈現比例關係對應之最佳損失發展因子為算術平均數，因此採用算術平均數計算損失發展因子及賠款準備金。

假設所有賠款在48個月已全部發展完成，故tail factor=1

計算LDF

LDF	12-24	12-36	36-48
2012	1.150	1.130	1.077
2013	1.200	1.083	
2014	1.100		
AVERAGE	1.150	1.107	1.077

賠款準備金

=1400*(1-1)+1300*(1.077-1)+1100*(1.107*1.077-1)+1000*(1.15*1.107*1.077-1)

=682

20.(2 分)

假設 A 保險公司於 2015 年底相關如下：

Accident Year	On-Level Premiums	Cumulative Paid Loss	Fitted Paid Emergence Pattern
2012	1,500,000	800,000	80%
2013	1,800,000	500,000	50%
2014	1,600,000	300,000	30%
2015	1,800,000	100,000	10%

Parameter standard deviation=500,000

Process variance/mean scale parameter(δ^2)=15,000

(1) 請採用 Cape Cod method 計算 A 保險公司於 2015 年底之賠款準備金。(1 分)

(2) 請計算第(1)小題賠款準備金之過程標準差(process standard deviation)。(0.5 分)

(3) 請計算第(1)小題賠款準備金之總標準差(total standard deviation)及變異係數(coefficient of variation)。(0.5 分)

【參考解答】

(1)

Total Premium*G(x)

=1,500,000*0.8+1,800,000*0.5+1,600,000*0.3+1,800,000*0.1=2,760,000

Total Loss

=800,000+500,000+300,000+100,000=1,700,000

ELR=1,700,000/2,760,000=61.6%

Accident Year	Premium	Expected Ultimate	Expected Emergence	Reserve
2012	1,500,000	924,000	20%	184,800
2013	1,800,000	1,108,800	50%	554,400
2014	1,600,000	985,600	70%	689,920
2015	1,800,000	1,108,800	90%	997,920
TOTAL				2,427,040

Eg. 2015: 1,800,000*0.616=1,180,800, 1,180,800*0.9=997,920

(2)

Process Variance =15,000*2,427,040
Process Stdev=(15,000*2,427,040)^0.5=190,803

(3)
Total Var = Process Var + Param Var
Total Var =500,000^2+190,803^2
Total Stdev=(500,000^2+190,803^2)^0.5=535,169
CV=535,169/2,427,040=22.1%

21.(2 分)
請依據下列資訊回答問題：

Portfolio	Portfolio Size ($000,000)	Length of Claim Run Off(in years)	Selected Coefficients of Variation(CoV)	
			Outstanding Claim Liability	Premium Liability
A	100	10	6%	y
B	500	10	x	8%
C	500	2	3%	2%

Event risk 不顯著且不考慮。

請分別選定 x 及 y 值，並依據 internally benchmarking independent risk 詳細說明其合理性。

【參考解答】

x:
Sample Answer 1
Since Portfolio B has a very long claim run-off time, the Premium Liability COV should be higher than the OCL COV. Moreover, Portfolio B is larger (in size) than Portfolio A, which is also having the same length of claim runoff years. Thus the OCL COV for A is larger than the OCL COV for B.
6% > x, 8% > x
In addition, OCL COV for B is longer than OCL COV for C, since they are the same size, and C has a much shorter runoff time than B

x > 3%
Select x = 5%
Sample Answer 2
6% - since the tail of claims matches A (C is a lot quicker, so lower CV), it would be an appropriate CV to account for the uncertainty.

y:
Sample Answer 1
PL COV(A) > PL COV(B) (since A is smaller than B, but with the same runoff period)
PL COV(A) > OCL COV(A) (more uncertainty for PL in long tail lines)
Y > 8% select y = 8.5%
Sample Answer 2
A has smaller size & longer runoff length than C that y should definitely be higher than 3%. Smaller book + same runoff length than B , y should be higher than 8%
I choose y to be 10% because it is longer tailed & smaller sized
Sample Answer 3
Y = 8% because this matches portfolio B which has a similar claim runoff length. Premium liability is risk that premiums written will not cover losses, and these two appear to write similar length (likely liability) coverage.

22.**(5 分)**
某精算人員在進行2個業務險別巨災風險損失相關性評估，並取得下列經驗資料：

	Loss($000,000)	
Year	LoB 1	LoB 2
2009	42	35
2010	40	44
2011	38	34
2012	36	32
2013	34	30
2014	30	29

(1) 請詳細說明並計算 3 個統計測度(statistical measure)可用來測量上表中 2 個業務險別之相關性。(3 分)
(2) 請評估第(1)小題中之測度，何者較適用於測量此 2 個業務險別之相關性。(2 分)

【參考解答】
(1)

Kendall's τ - measures relative orders and does not take into account the magnitudes of values in the lines of business being compared. Weights are determined by the number of swaps.

	Loss($000,000)				
Year	LoB 1	LoB 2	Rank 1	Rank 2	Swap 1
2009	42	35	1	2	1
2010	40	44	2	1	2
2011	38	34	3	3	3
2012	36	32	4	4	4
2013	34	30	5	5	6
2014	30	28	6	6	5

N=6, Q=1

Kendall's $\tau = 1 - [4Q/N \times (N - 1)] = 1 - [4(1)/6 \times (6 - 1)] = 0.8667$

Spearman's rank correlation - measures relative orders and does not take into account the magnitudes of values in the lines of business being compared.

	Loss($000,000)					
Year	LoB 1	LoB 2	Rank 1	Rank 2	Δrank	$(\Delta\text{rank})^2$
2009	42	35	1	2	1	1
2010	40	44	2	1	1	1
2011	38	34	3	3	0	0
2012	36	32	4	4	0	0
2013	34	30	5	5	0	0
2014	30	29	6	6	0	0

N=6, S=2

Spearman's $\rho = 1 - S \Big/ N(N^2 - 1)/6 = 1 - 2 \Big/ 6(6^2 - 1)/6 = 0.9429$

Pearson - a cardinal statistic weighting elements by the squares of their deviances from the mean.

Pearson $\rho = \text{cov}(x, y)/[\text{std}(x) \times \text{std}(y)] = 0.7196$

(2)

Ranks of preference for these measures is Kendall, Spearman, Pearson. Pearson is not

well suited to measuring dependency in thick tailed, skewed distributions. Both Kendall's Tau and Spearman's rank are ordinal measures, but Kendall's Tau has greater dependence in the tails. As the portfolio is exposed to Cats it is more reasonable to use Kendall's Tau to describe dependence..

23.(3 分)

某公司相關資訊如下：

- Return on equity (ROE): 12%
- Book value growth rate: 6%
- Discount rate: 9%
- 假設以上數值未來皆持續維持相同。
- 2014 年底該公司 book value 為$2,000,000.

(1) 採用 price-to-book value ratio 計算該公司於 2014 年底之價值(2 分)

(2) 請依據上述 ROE、Book value growth rate 及 Discount rate 之數值，評估假設這些數值未來持續維持相同之可能性(1 分)

【參考解答】

(1)

P / BV = 1 + [(ROE - k) / (k - g)]

= 1 + [(0.12 - 0.09) / (0.09-0.06)]

= 2

BV=2,000,000 * 2 = 4,000,000

(2)

It is not likely that the company will continue to see abnormal earnings indefinitely since as you continue to earn profit, this will attract more suppliers and prices will drop to remain competitive. This would lower the return until no abnormal earnings exist and returns are as expected. (Note that abnormal earnings is meant to represent earnings more than you your discount rate, your expected return)

24.(3 分)

A、B 兩家保險公司評估適當風險測度將資本分配至各險種，考慮的風險測度有下列四種:

Standard deviation

Semi-standard deviation

VaR

TVaR

(1) A 保險公司只在台灣承做住宅綜合保險及商業財產保險，該公司主要關注的風險為地震風

險，請評估 A 保險公司採用上述四種風險測度的適當性並決定最佳之風險測度。(1.5 分)

(2) B 保險公司只在台灣承做汽車保險，該公司主要關注的風險為定價適足性，請評估 B 保險公司採用上述四種風險測度的適當性並決定最佳之風險測度。(1.5 分)

【參考解答】

Solution 1

(1) Standard deviation: Not appropriate since it doesn't work well with skewed distributions like lines subject to EQ losses
Semi standard deviation: Not appropriate since it will end up ignoring part of the distribution, and won't highlight EQ risk enough (since it's total loss)
VaR - Not appropriate since its only one point, VaR(95) may not be high enough to capture EQ risks.
TVaR - While not ideal (I would use distortion of RTVaR) it's the best one in the list because it will average all loss values above VaR, better capturing EQ risk.

(2) Standard deviation: Not appropriate here because it builds in positive or beneficial deviations
Semi-standard: Best one since it uses the whole distribution, and focuses only on negative outcomes, even the small ones.
VaR an T-VaR: Not appropriate for two reasons:

a. Estimating at high levels ignores too much risk
b. Estimating at low levels puts too little weight on high risk (tail events)

Also, you would really want to look at the whole distribution here

Solution 2

(1) The standard deviation and semi-standard deviation are mean based risk measures they would not be appropriate in this case because for EQ risk you are more concerned with the tail. As EQ losses might be large but infrequent. VaR and TVaR would be more appropriate as they are tail risk measures. TVaR might be even more applicable because it measures the average loss in the tail not just the probability of experiencing a loss above a certain threshold like the VaR.

(2) To assess the risk of price inadequacy a mean-based risk measure would be more appropriate because it takes into account the entire loss distribution. Therefore I would use either the standard deviation or the semi-standard deviation. Semi-standard

might be slightly more appropriate as it focuses on only adverse outcomes which is what we are concerned about.

25.(3 分)

某保險公司債券投資資訊如下:

	Years to Maturity	Bond Rating	Credit Risk Solvency Capital Requirement ($000)	Bond Issuer's Current Annual Default Probability
Bond 1	1	BBB	100	1%
Bond 2	10	BBB	200	1%

假設最佳量化風險分散方式為採用相關係數=0.5.

(1) 依據上述債券投資內容，請說明 3 種該保險公司面臨之信用風險來源。(1.5 分)

(2) 假設上述 2 種債券來自同一發行者，請計算信用風險之資本要求。(1.5 分)

【參考解答】

(1)

Direct Default Risk - the bond issuer's may not make payments

Downgrade Risk - bonds could be downgraded a notch, lowering value of bond

Concentration risk in Bond rating BBB

(2)

Bonds are from the same issuer, so assume full correlation. Thus, capital requirement is sum of the individual capital requirements:100+200=300

26.(5 分)

某保險公司 2014 年底相關資訊如下：

Gross Premiums : $100 million

Gross Loss Ratio: 55%

Gross Expense Ratio: 35%

Risk-free interest rate :2%

Investment Yield: 5%

Beginning Surplus: $500 million

Probability of distress without reinsurance :5%

該保險公司考慮購買下列再保險合約：

Proposed reinsurance :30% quota share
Ceding Commission 20% of reinsurance premiums
Probability of distress with reinsurance 1%

假設所有保費、費用及佣金皆在年初給付，所有賠款皆在年底給付，且業務規模未來年度皆無改變。

(1) 採用 risk-adjusted present value of future earnings，計算購買再保險前後保險公司價值的差異。(3 分)

(2) 除第(1)小題方法外，請說明另外 2 種方法可用來評估保險公司風險轉移的價值 (quantifying the value of risk transfer to the insurance company.)。(2 分)

【參考解答】

(1)

D=(1-d)/(1+r)

Without Reinsurance

D=(1-0.05)/(1+0.02)=0.9314

Premium	100	
Loss	55	=100*0.55
Expense	35	=100*0.35
Income	10	=100-55-35

Equity	500	
Premium	100	
Expense	35	
Investable Assets	565	=500+100+35
Investment Income:	28.25	=565*0.05

K=1/D-1=1/0.9314-1=7.37%

PV=(10+28.25)/7.37%=519

With Reinsurance

D=(1-0.01)/(1+0.02)=0.9706

Gross Premium	100	
Net Premium	70	=100*0.7
Loss	38.5	=70*0.55

Expense	35	=100*0.35
Commission	6	=100*0.3*0.2
Income	2.5	=70-38.5-35+6

Equity	500	
Net Premium	70	
Expense	35	
Commission	6	
Investable Assets	541	=500+70+35+6
Investment Income:	27.05	=541*0.05

K=1/D-1=1/0.706-1=3.03%
PV=(2.5+27.05)/3.03%=975

value of the firm increases=975-519=456

(2)
Sample Solution 1
(a) Efficient Frontier Graph - which plot U/W profit against different risk measure such as, VaR, or TvaR. The reinsurance option in the upper left corner is preferred since it has higher U/W profit with low risk
(b) Cost Allocation Method - holding capital is not free, which required a return of capital. We compare the total cost of holding capital and reinsurance net cost, to see which option has lower cost

Sample Solution 2
(a) Simple Factor: Value = prob of distress × Equity
Difference with or without reinsurance quantifies the risk transfer
(b) Efficient Frontier Graph: the expected U/W profit against a risk measure. More U/W profit expected should be associated with a higher risk measures. The closest to the efficient frontier of the program is the more efficient we are

27.（ 4 分）

某險種 2015 年意外年度最終賠款之事前估計(*a priori* estimate)為 30 百萬元，分別採用 Chain Ladder 及 Bornhuetter-Ferguson 二種方法估計最終賠款，金額分別為 35 百萬元及 33 百萬元。

若改採 Benktander 法估計最終賠款，請計算其最終賠款及賠款準備金，並請詳列計算過程。

【參考解答】

$U_{GB} = (1-q_k)\, U_{CL} + q_k\, U_{BF}$

$= (1-q_k^2)\, U_{CL} + q_k^2\, U_0$

$35\cdot(1-q_k) + 33\cdot q_k = 35\cdot(1-q_k^2) + 30\cdot q_k^2$

$q_k = 0.4$

$C_k = U_{CL}\,(1-q_k) = 35\cdot 0.6 = 21$

$U_{GB} = 35\cdot(1-0.4) + 33\cdot 0.4 = 34.2$

$R_{GB} = U_{GB} - C_k = 34.2 - 21 = 13.2$

28.（4 分）

請簡述 Benktander 法相較於 Chain Ladder 及 Bornhuetter-Ferguson 二種方法的二個主要優點。

【參考解答】

(1) The Benktander method is found to have almost always a smaller mean squared error than the other two methods.

(2) The Benktander method is almost as precise as an exact Bayesian procedure.

29.（4 分）

已知下列資訊：

金額單位：千元

意外年度	滿期保費	預期損失率	最終損失發展因子	已付賠款
2013	300,000	50%	1.03	155,000
2014	360,000	52%	1.15	159,000
2015	390,000	55%	2.30	96,000

請採用 Benktander 方法計算 2013 至 2015 意外年度之最終賠款。

【參考解答】

意外年度	q_x	R_{BF}	U_{BF}	R_{GB}	U_{GB}

2013	0.029	4,369	159,369	4,642	159,642
2014	0.130	24,417	183,417	23,924	182,924
2015	0.565	121,239	217,239	122,787	218,787
合計		150,025	560,025	151,353	561,353

30.（4 分）

依據 Siewert, "A Model for Reserving Workers Compensation High Deductibles"，請計算下列高自負額勞工補償保險之最終賠款。

完整保障保費 - Full coverage premium	$356,000,000
完整保障預期損失率 - Full coverage expected loss ratio	55.50%
溢額比率 - Excess ratio (per-occurrence charge)	12.0%
集合比率 - Aggregate ratio (per-aggregate charge)	1.5%

【參考解答】

$P \cdot E \cdot \chi + P \cdot E \cdot (1-\chi) \cdot \varphi = 356,000,000 \cdot 55.5\% \cdot (12\% + (1-12\%) \cdot 1.5\%) = 26,317,656$