G5 高等準備金，價值評估與企業風險管理

1. **(3 分)**

A 保險公司有以下資訊:

累積損失金額(百萬元)

Accident Year	24 Months 已發生損失金額	最終損失金額
2011	12	18
2012	10	15
2013	16	22
2014	14	20
2015	23	

假設所有保單皆為一年期保單且在 1 月 1 日生效，危險暴露單位及承保範圍在 2011 年到 2014 皆相同。2015 年 1 月 1 日保單擴大承保範圍，該公司商品精算師評估影響如下:

- 擴大承保範圍造成損失金額增加20%。
- 最終賠款之80%預期會在24個月內報案，此報案比例之標準差為預期最終賠款之10%。
- AY 2015年最終損失之標準差為4百萬元。

(1)請使用 Bayesian Credibility Method 估計 AY 2015 年最終損失金額。**(2 分)**

(2)請說明 Least Squares Method 是否適合用來估計 AY 2015 年最終損失金額。**(1 分)**

【參考解答】

(1)

X= loss reported at 24 months

Y= Ultimate losses

L(x)=Z(x/d)+(1-Z)E[Y]

E(Y)= 1.2*(18+15+22+20)/4=22.5

VHM =((0.8)(4))^2=10.24

EVPV=0.1^2*[4^2+(1.2*(18+15+22+20)/4)^2]=5.223

Z=VHM/(VHM+EVPV)=10.24/(10.24+5.223)=0.662

L(x)=(0.662)(23/0.8)+(1-0.662)(22.5)=26.639 million

(2)

The least squares method is appropriate when the distribution of loss is not changing year over year. Given the coverage expansion and change in 2015 loss distribution, we cannot use the least squares method.

2. **(3 分)**

A 保險公司有汽車保險 2015 事故年度賠款於 2015 年底資訊如下:

- AY 2015年已付賠款:1百萬元
- 2015年滿期保費:6百萬元
- 初始預期損失率:60%
- 12-24個月損失發展因子:3.0
- 12-最終損失發展因子:4.0

(1) 請分別以 Chain ladder method, Bornhuetter-Ferguson method, Benktander method 計算 AY 2015 年最終損失金額。**(1.5 分)**

(2) 請計算 AY 2015 年於 2016 年之已付損失增量，使得 AY 2015 年於 2016 年底時，Benktander 最終賠款估計值比 Bornhuetter-Ferguson 最終賠款估計值多 100,000 元，假設所有損失發展選定值同第(1)小題。**(1.5 分)**

【參考解答】

(1)

CL: 1,000*4=4,000(K)

BF: 1,000+(1-1/4)*6,000*0.6=3,700(K)

Benktander:

q=1-1/4=0.75

R_{GB}=0.75*3,700=2,775

U_{GB}=1,000+2,775=3,775(K)

(2)

Let x= paid loss in 2015

24-Ult LDF=4/3=1.333

U_{BF}=(1,000+x)+(1-1/1.333)*6,000*0.6=1,900+x

[this is the losses at age 24 + expected losses * %unreported]

$R_{GB}=q_k U_{BF}$

q_k=1=1/1.333=0.25

R_{GB}=0.25*(1900+x)=475+0.25x

U_{GB}=(1,000+x)+(475+0.25x)=1,475+1.25x

$U_{GB}=U_{BF}$+100

1,475+1.25x=1,900+x+100

x=2,100(k)

3.（4分）

下列最終損失發展因子資訊為AY 2015年損失限額250,000元之資料，假設48個月損失完全發展完成:

Age	12	24	36	48
LDF	2.50	1.20	1.05	1.00

AY 2015年每一發展期間損失假設服從指數分配，平均數如下:

Age	12	24	36	48
平均數	20,000	40,000	50,000	60,000

假設AY年度損失趨勢為6%，CY年度損失趨勢為3%，AY2012~2015年損失資料如下:

AY	2012	2013	2014	2015
Paid to Date	15,000	12,500	10,000	6,000

請計算AY2012~2015年損失限額100,000元之最終損失金額。

【參考解答】

AY	AY TREND
2012	1.000
2013	1.060
2014	1.124
2015	1.191

CY TREND	12 Months	24 Months	36 Months	48 Months
2012	1.000	1.030	1.061	1.093
2013	1.030	1.061	1.093	1.126
2014	1.061	1.093	1.126	1.159
2015	1.093	1.126	1.159	1.194

TREND	12 Months	24 Months	36 Months	48 Months
2012	1.000	1.030	1.061	1.093
2013	1.092	1.125	1.158	1.193
2014	1.192	1.228	1.265	1.303
2015	1.301	1.340	1.381	1.422

Unlimit Mean	12 Months	24 Months	36 Months	48 Months
2012	15,367	30,735	38,419	46,102
2013	16,778	33,556	41,945	50,334
2014	18,318	36,637	45,796	54,955
2015	20,000	40,000	50,000	60,000

Eg. for AY 2014, age 12 Months: 18318=100000*1.192/1.301

LEV (X=100,000)	12 Months	24 Months	36 Months	48 Months
2012				40,834
2013			38,079	43,431
2014		34,246		46,048
2015	19,865			48,667

Eg. for AY 2015, age 12 Months: 19865=20000*[1-(e^(-100000/20000))]

LEV (B)	12 Months	24 Months	36 Months	48 Months
2012				
2013				
2014				
2015	20,000	39,923	49,663	59,070

Eg. for AY 2015, age 12 Months: 20000=20000*[1-(e^(-250000/20000))]

Finally, we can calculate the LDFs & Ultimate loss

AY	Paid to Date (1)	LDF (2)	Ult (3)=(1)*(2)
2012	13558	1.0000	13558
2013	12540	1.0069	12626
2014	9775	1.0905	10660
2015	5590	2.0737	11592

Eg. for AY 2015, LDF: 2.0737=48667/19865

4. **(3 分)**

A保險公司損失資料如下:

Accident Year	Incremental Incurred Losses($000) 12 Months	24 Months	36 Months	Earned Premium
2013	22,000	11,000	4,000	54,000
2014	26,000	10,000		56,000
2015	20,000			58,000

(1) 請採用 Neuhaus Credibility weight 以R^{ind}及R^{coll}之線性組合估計 AY 2015 年未付賠款金額。**(1.5 分)**

(2) 假設$\text{Var}(U_i) = \text{Var}(U_i^{BC})$，請採用 Hurlmann Method 在最佳可信度及最小變異數前提下，估計 AY 2015 年未付賠款金額。**(1.5 分)**

【參考解答】

(1)

m1 = 68,000 / 168,000 = 0.405

m2 =21,000 / 110,000 = 0.191

m3 = 4,000 / 54,000 = 0.074

mtot = 0.405 + 0.191 + 0.074 = 0.670

p1 = m1 / mtot = 0.405 / 0.670 = 0.604

q1 = 1 - p1 = 0.396

Rind = q1 / p1 × C1,3

= 0.396 / 0.604 ×20,000 =13,093

Rcoll = q1 × UBC

= q1 × mtot × V1

= 0.396 × 0.670 × 58,000 = 15,369

ZWN = m1 = 0.405

RC = ZWN × Rind + (1 - ZWN) × Rcoll

= 0.405 ×13,093 + (1 - 0.405) ×15,369 = 14,448

(2)

Z* = p1 / (p1 + $\sqrt{}$p1) = 0.604 / (0.604 + $\sqrt{}$0.604) = 0.437

RC = Z* × Rind + (1 - Z*) × Rcoll = 0.437 ×13,093 + (1 - 0.437) ×15,369 = 14,374

5. (3 分)

依據下列資料，請測試並說明相鄰之損失發展因子是否存在相關性。

Cumulative Paid Losses

Accident Year	12 Months	24 Months	36 Months	48 Months	60 Months	72 Months
2010	8,000	10,000	11,000	11,500	11,800	12,000
2011	14,000	20,000	21,500	22,500	23,000	
2012	12,000	18,000	20,000	22,000		
2013	15,000	19,000	22,000			
2014	11,000	22,000				
2015	14,000					

【參考解答】

LDF

	12-24	24-36	36-48	48-60	60-72
2010	1.250	1.100	1.045	1.026	1.017
2011	1.429	1.075	1.047	1.022	
2012	1.500	1.111	1.100		
2013	1.267	1.158			
2014	2.000				

12-24 & 24-36

12-24 LDF rank	12-24 LDF rank	Difference	Squared
4	3	1	1
2	4	2	4
1	2	1	1
3	1	2	4

S=1+4+1+4=10

$$T_k = 1 - \frac{s}{(n*(n^2-1)/6)}$$

T_{24}=1-(10/(4*15)/6)=0

24-36 & 36-48

24-36 LDF rank	36-48 LDF rank	Difference	Squared
2	3	1	1
3	2	1	1
1	1	0	0

S=1+1=2

T_{36}=1-(2/(3*8)/6)=0.5

36-48 & 48-60

36-48 LDF rank	48-60 LDF rank	Difference	Squared
2	1	1	1
1	2	1	1

S=1+1=2

T_{36}=1-(2/(2*3)/6)=-1

T=(0*3+0.5*2+(-1)*1)/6=0

VAR(T)=1/((6-2)*(6-3)/2)=0.167

we use a threshold of 50%, which is the percentile range [25%, 75%]. Thus, the confidence interval is

CI=(-0.67*0.167^0.5, 0.67*0.167^0.5)=(-0.274,0.274)

The test statistic T= 0 is within the confidence interval. Therefore, we do not reject the Null Hypothesis that the adjacent LDFs are uncorrelated.

6. (3 分)

依據下列資料並採用 90%信賴區間，請測試並說明是否存在曆年度趨勢現象。

Cumulative Paid Losses

Accident Year	12 Months	24 Months	36 Months	48 Months	60 Months	72 Months
2010	12,000	20,000	24,000	25,500	26,800	27,000
2011	18,000	20,000	25,000	27,500	29,000	
2012	12,000	18,000	21,000	23,000		
2013	15,000	25,000	28,000			
2014	10,000	22,000				
2015	15,000					

【參考解答】

LDF

AY	12-24	24-36	36-48	48-60	60-72
2010	1.667	1.200	1.063	1.051	1.007
2011	1.111	1.250	1.100	1.055	
2012	1.500	1.167	1.095		
2013	1.667	1.120			
2014	2.200				

Rank columns, calculate z=min(S,L) for each diagonal

AY	12-24	24-36	36-48	48-60
2010	*	L	S	S
2011	S	L	L	L
2012	S	S	*	
2013	L	S		
2014	L			

（若將 2010 AY 12-24 視為 L 及 2013 AY 12-24 視為*且對應之計算結果皆正確亦給分）

Diagonal	n	m	c_n	$E[z_n]$	$Var[z_n]$	z
1	0	0				
2	2	0	0.5	0.5	0.25	1
3	3	1	0.75	0.75	0.188	1
4	4	1	0.75	1.25	0.438	2
5	3	1	0.75	0.75	0.188	1

Sum of $E[z_n]$=0.5+0.75+1.25+0.75=3.25

Sum of $Var[z_n]$=0.25+0.188+0.438+0.188=1.064

Z=1+1+2+1=5

The Confidence Interval is $3.25 \pm 1.645 * \sqrt{1.064}$=(1.55,4.95)

Since Z=5, and it is not within the confidence interval, there are calendar year effects in the triangle.

7. (4 分)

依據下列資料:

	Cumulative Paid Losses($000)		
Accident Year	12 Months	24 Months	36 Months
2013	2,500	4,100	5,000
2014	2,600	4,000	
2015	2,800		

假設AY loss emergence pattern (growth function)服從Weibull 分配，

Weibull:$G(x|\omega,\theta) = 1 - \exp(-(x/\theta)^{\omega})$，其中$\omega = 1.5$，$\theta = 20$

(1) 假設採用 LDF method 計算賠款準備金，請計算 AY 2013~2015 年賠款準備金之過程標準差(process standard deviation)。(2.5 分)

(2) 請畫出 normalized residuals plotted against the increment age of loss emergence，並依據你畫出的圖形評論採用 Weibull 分布的合適性。(1.5 分)

【參考解答】

(1)

We need σ^2. We have:

n=6

p= 5; one for each row and one for each column (except the first one)

Check for truncation at twice the age of the triangle:

G(66)= 0.998

Very close to 1.000; thus no truncation is necessary.

G(6)= 0.152

G(18)= 0.574

G(30)= 0.841

Ultimate Losses:

2013:5000/0.841=5945

2014:4000/0.574=6969

2015:2800/0.152=18421

Reserve=(5945-5000)+(6969-4000)+(18421-2800)=19535

Expected Cumulative Loss: G(x)*Ult. loss

Accident Year	12 Months	24 Months	36 Months
2013	904	3,413	5,000
2014	1,059	4,000	
2015	2,800		

Expected Incremental Loss

Accident Year	12 Months	24 Months	36 Months
2013	904	2,509	1,587
2014	1,059	2,941	
2015	2,800		

Actual Incremental Loss

Accident Year	12 Months	24 Months	36 Months
2013	2,500	1,600	900
2014	2,600	1,400	
2015	2,800		

$$\sigma^2 = \frac{1}{6-5}\left[\frac{(2500-904)^2}{904} + \frac{(1600-2509)^2}{2509} + \cdots + \frac{(2800-2800)^2}{2800}\right] = 6495$$

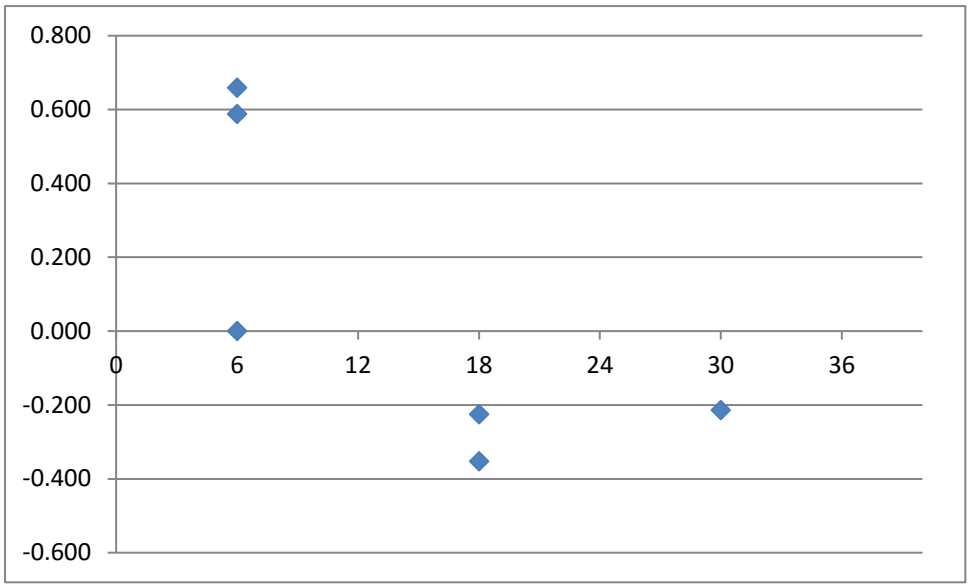
Process Standard Deviation: (6495*19535)^0.5=11264

(2)

Normalized Residuals

Accident Year	12 Months	24 Months	36 Months
2013	0.659	-0.225	-0.214
2014	0.587	-0.353	
2015	0.000		

Ex:0.659=(2500-904)/(6495*904^0.5)



This estimate of loss emergence is very poor. The residuals are positive for age 6, and negative for ages 18 and above. We expect a random fluctuation around zero, not this decreasing pattern.

8. (3 分)

請依據下列資訊回答問題：

Portfolio	Portfolio Size ($000,000)	Length of Claim Run Off(in years)	Selected Coefficients of Variation(CoV)	
			Outstanding Claim Liability	Premium Liability
A	200	15	8%	y
B	600	15	x	10%
C	600	3	4%	3%

Event risk 不顯著且不考慮。

請分別選定 x 及 y 值，並依據 internally benchmarking independent risk 詳細說明其合理性。

【参考解答】

x:

Since Portfolio B has a very long claim run-off time, the Premium Liability COV should be higher than the OCL COV. Moreover, Portfolio B is larger (in size) than Portfolio A, which is also having the same length of claim runoff years. Thus the OCL COV for A is larger than the OCL COV for B.

8% > x, 10% > x

In addition, OCL COV for B is longer than OCL COV for C, since they are the same size, and C has a much shorter runoff time than B

x > 4%

Select x = 6%

y:

Sample Answer 1

PL COV(A) > PL COV(B) (since A is smaller than B, but with the same runoff period)

PL COV(A) > OCL COV(A) (more uncertainty for PL in long tail lines)

Y > 10% select y = 11%

Sample Answer 2

A has smaller size & longer runoff length than C that y should definitely be higher than 4%. Smaller book + same runoff length than B , y should be higher than 10%

I choose y to be 12% because it is longer tailed & smaller sized

Sample Answer 3

Y = 10% because this matches portfolio B which has a similar claim runoff length. Premium liability is risk that premiums written will not cover losses, and these two appear to write similar length (likely liability) coverage.

9.（4 分）

某公司相關資訊如下：

- 投資人要求之Return on equity (ROE): 9%

單位:元

	2016年	2017年	2018年
Beginning GAAP Equity	120,000	125,000	132,000
Net Income	12,000	14,000	16,000
Minimum Capital Requirement	125,000	132,000	137,000

(1) 假設每年異常報酬成長率永久維持=3%，請計算該公司於 2016 年初之價值。(1.5 分)

(2) 假設每年異常報酬成長率於預測期間結束後 3 年=3%且 2022 年之後皆=0，請計算該公司於 2016 年初之價值。(1.5 分)

(3) 請評論上述 2 種情境假設何者較符合實際。(1 分)

【參考解答】

(1)

AE2016 = 12,000 - 120,000 * 0.09 = 2,000

AE2017 = 14,000 - 125,000 * 0.09 = 2,750

AE2018 = 16,000 - 132,000 * 0.09 = 4,120

V0 = BV0 + ΣAE/(1+k)i + Terminal Value

= 120,000 + 2,000/1.09 + 2,750/1.09^2 + 4,120/1.09^3 + [4,120*1.03/(0.09-0.03)]/1.09^3

= 181,211

(2)

V0 = 120,000 + 2,000/1.09 + 2,750/1.09^2 + 4,120/1.09^3 + 4,120*1.03/1.09^4 + 4,120*1.03^2/1.09^5 + 4,120*1.03^3/1.09^6

= 135,128

(3)

第(2)項is more realistic since maintaining abnormal earnings in perpetuity is not realistic in practice.

10（4 分）

依據下列 2 間台灣保險公司資料回答問題：

・A保險公司只承保任意汽車保險且投資標的為存續期間與負債相同之台灣政府公債。

・B保險公司只承保multi-line commercial insurance且投資標的包含多種固定收益證卷及股票市場。

請依據下列風險項目，評析兩家公司面臨之問題。

(1) Pricing Risk（1 分）

(2) Claim Variability(1 分)

(3) Market Risk(1 分)

(4) Correlation(1 分)

【参考解答】

(1)

A保險公司: This line of business tends to be competitive, putting strong pressure on lower prices. The pricing risk is that prices will be set to low, so at to hit volume targets.

B保險公司: These accounts are not homogenous, and thus it is more difficult to get the pricing right. Also, if some of the business is long-tailed, it could be years before the insurer realizes it is underpriced.

(2)

A保險公司: Not a major risk, since this book is a high frequency, moderate severity book. Some items that could affect claim variability are inflation, changes in law or judicial perspective.

B保險公司: These policies tend to be high limits' policies, and thus the insurer has significant claim variability risk. The lack of homogeneity among clients makes this worse.

(3)

A保險公司: Since the Assets are liability matched to the liabilities, much of the market risk is hedged. There is still a minor risk that the bonds don't pay, or possibly fall in value. The largest risk here is that the duration calculation was
not done correctly, and thus the ALM match is off

B保險公司: Company is invested in equities and corporate bonds, both of which have significant market risks, which could significantly impact the balance sheet in a short period of time.

(4)

A保險公司: Correlation between the business (auto) and the assets (Taiwan Bonds) here is negligible.

B保險公司: If the company has investments in any of it's clients, the client could have an event that reduces the value of those investments, at the same time as an insured loss. More likely though, a downturn in the economy would
impact the prices of both the bonds and the equities the insurer is holding. This same downturn may lead the client to require less insurance, reducing the premium of the insurer.

11.(4 分)

根據以下至 2015 年之資訊回答下列問題

意外年度	平準保費 (on-level earned premium)	Growth Function	已發生賠款
2013	20,000	79.50%	10,500
2014	18,000	60.50%	6,500
2015	17,000	15.00%	1,800
總數	55,000		18,800

針對預估未付賠款之係數標準差(parameter standard deviation)為 800

意外年度之預期損失發展的 growth function 為 loglogistic function

$G(x \mid \omega, \theta) = x^{\omega} / (x^{\omega} + \theta^{\omega})$ ， ω=1.956， θ=15.286，

x 為平均意外發生日至評估日之月數

損失增量之發展乃根據 over-dispersed Poisson 分配，其中 scaling factor σ^2 = 9

(1)(2.5 分)

請採用 Cape Cod 法，並以五年為 truncation point 預估以上所有意外年度之未付賠款。

(2)(1.5 分)

請計算(1)之未付賠款的標準差(standard deviation)。

【參考解答】

(1)

Growth function for truncation = $x^{\omega} / (x^{\omega} + \Theta^{\omega}) = 54^{1.956} / (54^{1.956} + 15.286^{1.956}) =$ 0.922

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9) = (1) * ELR * (8)
意外年度	平準保費 (on-level earned premium)	Age at 12/31/2015	Average Age	Growth Function	= (1) *(4)	已發生賠款	ELR = (6)/(5)	=0.922 - (4)	預估未付賠款
2013	20,000	36	30	79.50%	15,900	10,500		12.7%	1,628
2014	18,000	24	18	60.50%	10,890	6,500		31.7%	3,658
2015	17,000	12	6	15.00%	2,550	1,800		77.2%	8,412
總數	55,000				29,340	18,800	64.1%		13,698

(2) Process variance = Mean * σ^2 = 13698 * 9 = 123,282

Total Variance = Process variance + Parameter variance = 123282+800^2 = 763,282

Standard deviation = (763,282)^(1/2) = 873.7

12.(5 分)

在 Chain Ladder 方法中一個主要的假設為任一意外年度之損失與其他意外年度之損失為獨立關係，請根據以下損失發展三角形與損失發展因子回答下列問題：

	已發生賠款				
意外年度	12個月	24個月	36個月	48個月	60個月
2011	3,500	4,900	5,782	6,245	6,307
2012	3,200	4,160	4,992	5,242	
2013	3,085	4,165	4,789		
2014	2,880	3,600			
2015	2,780				

	損失發展因子				
意外年度	12個月	24個月	36個月	48個月	
2011	1.40	1.18	1.08	1.01	
2012	1.30	1.20	1.05		
2013	1.35	1.15			
2014	1.25				

(1)請以 95%的信賴區間測試虛無假設(null hypothesis)上方之損失發展三角形不具顯著的 calendar year effect (3 分)

(2)請描述 Chain Ladder 方法的其他兩個假設(2 分)

【參考解答】

(1)

95% 信賴區間，statistic = 1.96

	12-24	24-36	36-48
中位數(medium)	=(1.3+1.35)/2 = 1.325	1.18	=(1.08+1.05)/2=1.065

將損失發展因子大於中位數標示為 L，小中位數標示為 S

	損失發展因子		
意外年度	12個月	24個月	36個月
2011	L	X	L
2012	S	L	S
2013	L	S	
2014	S		

$E(Z_j) = n/2 - \binom{n-1}{m} \times (n/2^n)$

$Var(Z_j) = n \times (n-1)/4 - \binom{n-1}{m} \times (n \times (n-1)/2^n) + E(Z) - E(Z)^2$

$n = S + L$

$m = (n-1)/2$

$Z = \min(S, L)$

Let j = Diagonal

J	N	m	Z	E(Z)	Var(Z)
1	1	0	0	0	0
2	1	0	0	0	0
3	3	1	0	0.75	0.1875
4	3	1	0	0.75	0.1875
			0	1.50	0.375

信賴區間 = 1.5 +- 1.96 * (0.375)^0.5 = (0.3,2.7)

Z=0 不在信賴區間內，因此否決了虛無假設，所以損失發展三角形不具顯著的 calendar year effect

(2)

a. 下一段發展期間之預期損失發展增量等於累積至此發展期間之損失乘以損失法展因子

b. 下一段發展期間之預期損失 variance 與累積至此發展期間之損失 variance 呈現正比關係

13.(2 分)

(1) 請說明三個造成再保險公司報案的時間比簽單公司更久 (longer reporting lag)的原因。(1 分)

(2) 請說明兩個為何再保險公司無法採用保險產業損失法展統計資料預估損失發展之趨勢。(1 分)

【參考解答】

(1) a. 賠案必須先報至簽單公司，再透過簽單公司報給再保險公司。

b. 若簽單公司低估賠款金額，簽單公司將會延遲報案給再保險公司。

c. 在侵權相關的賠案通常追償花較長的時間，且須等待法院的判決損失發生日期。

(2) a. 延遲報案並且損失發展時間較長。

b. 再保險公司在相同險種下承保的風險有極大的差異，因此其他公司的統計資料無法適用。

14.(5 分)

根據以下某再保險公司截至 12/31/2015 為止之資料回答下列問題：

Calendar/ Accident Year	Earned Risk Pure Premium	Adjusted Premium	Aggregate Reported Loss	Reported loss lag	Chain Ladder IBNR
2011	12,500	13,000	11,500	95%	500
2012	13,300	13,800	10,000	85%	1,500
2013	14,000	14,200	8,500	72%	3,100
2014	15,500	15,800	7,900	55%	4,500
2015	16,000	16,000	6,000	40%	7,000
總數	71,300	72,800	43,900		16,600

(1)請採用 Standard Buhlmann 方法計算所有意外年度之 IBNR。(1 分)

(2)請分別描述若再保人採用 Standard-Buhlmann 方法計算 IBNR 之優點與缺點。(2 分)

(3)請採用 credibility-weighted estimate 的方法結合 chain ladder 法和 Standard-Buhlmann 法計算 IBNR，其中 chain ladder 法的 credibility factor 為 0.4。(2 分)

【參考解答】

(1) SB ELR = 43900/(13000*95%+13800*85%+14200*72%+15800*55%+16000*40%) = 88.9%

SB IBNR = 88.9% * (13000*5%+13800*15%+14200*28%+15800*45%+16000*60%) = 20,808

(2) 優點：採用實際損失資料計算 ELR，而非如 BF 法採用主觀判斷。

缺點：須調整每年的保費以反映過去的費率變更。

(3) Credibility-weighted IBNR =

(0.4*95%*500+(1-0.4*95%)*578+0.4*85%*1500+(1-0.4*85%)*1840+0.4*72%*3100+(1-0.4*72%)*3535+0.4*55%*4500+(1-0.4*55%)*6321+0.4*40%*7000+(1-0.4*40%)*8534)

= 19,891

15.(5 分)

請根據至 12/31/2015 為止之資料，依照 Sahasrabuddhe 文章中所述之方法，以意外年度 2015 年的成本水準為基礎，計算已發生損失發展三角形的最新對角線值的 basic limit of loss。

	已發生賠款			
意外年度	12個月	24個月	36個月	48個月
2012	333,000	612,000	650,000	700,000
2013	314,000	570,000	600,000	
2014	352,000	640,000		
2015	365,000			

上方之損失發展三角形是根據 unlimited basis

在 unlimited basis 下，歷年之損失發展趨勢為每年 4%

Basic Limit = 50,000

Exponential distribution 可以滿足理賠損失情況下，對於意外年度 2015 年之損失發展期間產生以下之 unlimited claim size mean：

	12	24	36	48
Unlimited Claim Size Mean	36,000	57,000	64,000	72,000

Mean of exponential distribution: θ

Variance of exponential distribution: θ^2

Limited mean of exponential distribution at limit K: $\theta(1-e^{(\frac{-k}{\theta})})$

【參考解答】

	Triangle of Trend			
意外年度	12個月	24個月	36個月	48個月
2012	1.000	1.040	1.082	1.125
2013	1.040	1.082	1.125	1.170
2014	1.082	1.125	1.170	1.217
2015	1.125	1.170	1.217	1.265

	Triangle of Trended θ			
意外年度	12個月	24個月	36個月	48個月
2012	32,004	50,673	56,896	64,008
2013	33,284	52,700	59,172	
2014	34,615	54,808		
2015	36,000	57,000	64,000	72,000

AY 2012 at 12 months = 36000*(1/1.125) = 32004

	Limited Expected Value for basic limit			
意外年度	12個月	24個月	36個月	48個月
2012	27,023	33,291	34,699	36,047

↓

= 36000*(1-e^(-50000/36000)) = 27023

AY Cumulative loss at basic limit cost level

2012 = 700,000*(36,047/64,008) = 394,215

2013 = 600,000*(34,699/59,172) = 351,845

2014 = 640,000*(33,291/54,808) = 388,743

2015 = 365,000*(27,023/36,000) = 273,983

16.(2 分)

請說明四個應用 over-dispersed Poisson bootstrap model 所面臨資料上問題，並分別提出每個問題的調整方式。

【參考解答】

(1) 發展之損失增量為負值，調整方式將負值調整為零

(2) 不完整的質或數據，調整方式為選擇附近的數值代替

(3) 承保之風險在過去幾年變化很大，調整資料以獲得純保費，並以此按照承保風險變化程度進行調整。

(4) Heteroecthesious data/misshapen data/partial year/interim evaluation dates: project future incremental values before applying model; need to annualize then
de-annualize results

17.(3 分)

請根據以下保險公司之財務資訊，依 abnormal earnings valuation model 計算公司於 1/1/2017 之價值

	2017	2018	2019
期初股東權益 (Equity)	1,000,000	1,090,000	1,190,000
淨損益 (Net Income)	150,000	166,000	183,000
年底支付股利 (Dividend)	60,000	66,000	73,000

- 預估 equity market risk premium = 8%
- 無風險利率(risk-free rate) = 2%
- 保險公司之 beta（β）= 1.2
- 公司計畫維持目前之股利 payout ratio = 60%
- 採用 CAPM 決定風險調整後之折現率(risk-adjusted discount rate)
- 假設 abnormal earning 在以上預估期間之後會逐年平均下降至 2024 年時降為零

【參考解答】

Required return = 2% + 1.2 * 8% = 11.6%

Abnormal earning in 2017 = 150,000 - 11.6% * 1,000,000 = 34,000

Abnormal earning in 2018 = 166,000 - 11.6% * 1,090,000 = 39,560

Abnormal earning in 2019 = 183,000 - 11.6% * 1,190,000 = 44,960

	abnormal earnings		PV of abnormal earnings
年	預估期間	逐年遞減期間	
2017	34,000		=34000/(1.116)^1=30466
2018	39,560		=39560/(1.116)^2=31763
2019	44,960		=44960/(1.116)^3=32347
2020		= 44960 *4/5 = 35,968	=35968/(1.116)^4=23188
2021		= 44960 *3/5 = 26,976	=26967/(1.116)^5=15578
2022		= 44960 *2/5 = 17,984	=17984/(1.116)^6=9309
2023		= 44960 *1/5 = 8,992	=8992/(1.116)^7=4171
2024		0	0
總數			146,822
期初股東權益			1,000,000
公司價值			1,146,822

18.(3 分)

請根據以下追溯費率之保險合約之資訊，計算 Retrospective premium asset。

以下為三個 retrospective adjustment 相關數據：

Retro Adjustment Period	% Loss Emerged	Loss Capping Ratio
第一	78%	0.92
第二	25%	0.75
第三	8%	0.56

- Basic premium factor = 0.25
- Loss conversion factor = 1.3
- Tax multiplier = 1.05
- 預期 unlimited loss = 300,000
- 預期損失率 = 75%
- 目前帳上之保費= 350,000
- 假設沒有任何保險合約已進行第一次 retro adjustment

【參考解答】

Cumulative capping ratio = 78%*0.92+25%*0.75+8%*0.56 = 95%

Capped loss = 300,000*95% = 285,000

Standard premium = 300,000/0.75 = 400,000

Premium asset = (0.25*400,000+1.3*285,000)*1.05-350,000 = 144,025

19.(2 分)

某保險公司所持有之債券如下：

(1) A 債券由一家 credit rating 為 AA 之美國銀行所發行，價值為 10 億美元，平均到期日為五年。

(2) B 債券由一家 credit rating 為 BBB 之美國軟體公司所發行，價值為 15 億美元，平均到期日為十年。

此保險公司並未持有除了 A 與 B 之外的債券。

請說明三個對於保險公司信用風險具影響之主因，並說明這三個主因與公司所持有債券之關係。

【參考解答】

(1) 債券之信用品質

根據信用品等，軟體公司所發行之債券的信用風險較高。

(2) 到期時間

平均到期時間愈長，信用風險愈高；因此軟體公司所發行之債券的信用風險較高。

(3) 集中度（類型、產業、區域之集中度）

A 和 B 債券來自兩個關聯性不高的產業，保險公司藉此降低了集中度風險。

20.(3 分)

請根據以下資料依 Dividend Discount Model 計算保險公司之價值。

預期市場保險產業之 ROE = 12%

無風險利率 = 2%

保險公司之 beta（β）= 0.8

此保險公司之 plowback ratio = 70%

採用 CAPM 決定風險調整後之折現率(risk-adjusted discount rate)

	實際數	預估數				
	2011	2012	2013	2014	2015	2016
淨損益	180	190	200	211	222	234
期初股東權益	2,000	2,108	2,222	2,342	2,468	2,602
期末股東權益	2,108	2,222	2,342	2,468	2,602	2,742

【參考解答】

Dividend	76.0	80.0	84.4	88.8	93.6
ROE = Net Income /Beginning Equity	9%	9%	9%	9%	9%

g = plowback ratio * ROE = 70% * 9% = 6.3%

discount rate = 2% + 0.8 * (12%-2%) = 10%

公司價值 = $\frac{76}{1.1}+\frac{80}{1.1^2}+\frac{84.4}{1.1^3}+\frac{88.8}{1.1^4}+\frac{93.6}{1.1^5}+\frac{93.6\times1.063}{1.1^5\times(0.1-0.063)}$ = 1,987.1

21.(4 分)

已知下列資訊：

意外年度	36 個月 已發生賠款	最終賠款
2010	275,000	289,000
2011	305,000	315,000
2012	290,000	313,000
2013	327,000	342,000
2014	338,000	

某精算人員分別以鏈結比率法(Link ratio approach)、預算損失法(Budgeted loss approach)、Bornheutter-Ferguson 法及最小平方法(Least-squares approach)估計 2014 意外年度最終賠款。

(1)Bornheutter-Ferguson 法及最小平方法皆為鏈結比率法及預算損失法的可信度加權平均值，請分別計算 Bornheutter-Ferguson 法及最小平方法的可信度值(Z)。(2 分)

(2)請以最小平方法計算 2014 意外年度最終賠款。(2 分)

【參考解答】

(1)

$\overline{X}$ = 299,250

$\overline{Y}$ = 314,750

$\overline{XY}$ = 94538500000

$\overline{X^2}$ = 89919750000

b=$\frac{\overline{XY}-\overline{XY}}{\overline{X^2}-\overline{X}^2}$ = 0.947

a=$\overline{Y}-b\overline{X}$= 31407.313

c=$\overline{Y}/\overline{X}$= 1.052

Z^{LS}=b/c= 0.900

Z^{BF}=1/c= 0.951

(2) Y_2014=a+bX= 351,440

22.（3 分）

2016 意外年度已知已付賠款為$25,000,000，事前估計最終賠款為$34,000,000，某精算人員採 Benktander 法估計最終賠款，計算結果為$33,375,000。

請改採鏈梯法(Chain-Ladder approach)計算 2016 意外年度最終賠款。

【參考解答】

(33,375-25,000)=q(25,000+34,000q)

q=0.25

$Unpaid^{CL}$ = 25,000*0.25/(1-0.25) = 8,333.33

23.(3 分)

已知某公司及市場下列資訊：

預估 2017 年獲利	1.86 億元
該公司 ROE	8%
該公司 beta	1.1
該公司再投資比率 (plowback ratio)	30%
無風險利率	1.20%
預期市場報酬率	5.50%

請採用股息折現法(Dividend discount model)計算 2016 年底之該公司價值。

【參考解答】

預估 2017 年現金股利 = 186*(1-30%) = 130.2

K = 1.2%+1.1(5.5%-1.2%) = 5.93%

g = 8%*30% = 2.4%

V_0 = 130.2/(5.93%-2.4%) = 3,688.385

24.(5 分)

依據下列資訊：

金額單位：千元

意外年度	滿期總保費	滿期純保費	調整純保費	已報賠款	預期損失率	已報賠款延遲
2012	365,000	245,000	260,000	203,000	55%	95%
2013	383,000	264,000	270,000	191,000	57%	85%
2014	392,000	268,000	272,000	169,000	57%	70%
2015	395,000	270,000	273,000	133,000	61%	50%
2016	425,000	289,000	289,000	98,000	59%	30%

請分別以鏈梯法(Chain-Ladder)、Bornhuetter-Ferguson 法及 Stanard-Buhlmann 法計算未報賠款。

【參考解答】

鏈梯法：

203000(1/.95-1)+191000(1/.85-1)+169000(1/.70-1)+133000(1/.50-1)+98000(1/.30-1)

= 478485

Bornhuetter-Ferguson 法：

365000*55%*(1-.95)+383000*57%*(1-.85)+392000*57%*(1-.70)+395000*61%*(1-.50)+425000*59*(1-.30)

= 405816

Stanard-Buhlmann 法：

ELR = (203+191+169+133+98)/(260*.95+270*.85+272*.7+273*.5+289*.3) = 89.2%

IBNR = 89.2%*(260*(1-.95)+270*(1-.85)+272*(1-.7)+273*(1-.5)+289*(1-.3))

= 422719

25.（5 分）

已知某險種鏈結比率如下：

	12-24	24-36	36-48	48-60	60-72
2011	1.695	1.296	1.121	1.046	1.022
2012	1.641	1.319	1.118	1.058	
2013	1.707	1.362	1.124		
2014	1.688	1.315			
2015	1.747				

請依據 Mack 所述方法，測試在 90%信心水準下該險種損失三角形是否具有曆年效應(calendar year effects)。

(常態分配 90 百分位之 z 值為 1.645。)

【參考解答】

2011	*	S	*	S	*
2012	S	L	S	L	
2013	L	L	L		
2014	S	S			
2015	L				

Diagonal j	Sj	Lj	Nj	mj	Zj	E(Zj)	Var(Zj)
2	2	0	2	0	0	0.5	0.25
3	0	2	2	0	0	0.5	0.25
4	3	1	4	1	1	1.25	0.4375
5	1	3	4	1	1	1.25	0.4375
					2	3.5	1.375

90% confidence interval: $(3.5-1.645\sqrt{1.375}, 3.5+1.645\sqrt{1.375}) = (1.571, 5.429)$

因 2.0 在(1.571, 5.429)範圍內，測試結果無曆年效應。

26.（3 分）

某再保公司精算人員評估採用鏈梯法(Chain-ladder method)、Bornheutter-Ferguson 法及 Stanard-Buhlmann 法估計未報賠款，請分別敘述三種方法之缺點為何。

【參考解答】

(1) 鏈梯法：

It is dependent on development patterns from the historical data. If major changes in the claims department have taken place only in the most recent Accident Years/Calendar Years, these changes will not be considered in the LDFs calculated from historical data.

(2) Bornheutter-Ferguson:

Expected loss ratio is selected somewhat arbitrarily and reserve estimate depends on this selection.

(3) Stanard-Buhlmann:

It is sometimes difficult to obtain current rate level premium for older years of data.

27. (3 分)

請依據 International Actuarial Association, "A Global Framework for Insurer Solvency Assessment"，請簡述核保風險的 8 項主要風險來源。

【參考解答】

- Underwriting Process Risk- risk from exposure to financial losses related to the selection and approval of risks to be insured
- Pricing Risk- risk that the prices charged by the company for insurance contracts will be ultimately inadequate to support the future obligations arising from those contracts
- Product Design Risk- risk that the company faces risk exposure under its insurance contracts that were unanticipated in the design and pricing of the insurance contract
- Claims Risk (for each peril)- risk that many more claims occur than expected or that some claims that occur are much larger than expected claims resulting in unexpected losses. This includes both the risk that a claim may occur, as well as the risk that the claim might develop adversely after it occurs
- Economic Environment Risk- risk that social conditions will change in a manner that has an adverse effect on the company
- Net Retention Risk- risk that higher retention of insurance loss exposures results in losses due to catastrophic or concentrated claims experience
- Policyholder Behaviour Risk- risk that the insurance company"s policyholders will act in ways that are unanticipated and have an adverse effect on the company
- Reserving Risk – risk that the provisions held in the insurer"s financial statements for its policyholder obligations (also “claim liabilities,” “loss reserves” or “technical provisions”) will prove to be inadequate.

28.（2 分）

依據 Patrik, "Reinsurance"，再保險賠款準備金評估相較一般簽單公司較為困難，請簡述其中主要的六項技術問題。

（僅須回答其中六項）

【參考解答】

- Claim report lags to reinsurers are generally longer, especially for casualty excess losses.
- There is a persistent upward development of most claim reserves.
- Claims reporting patterns differ greatly by reinsurance line, by type of contract and specific contract terms, by cedant, and possibly by intermediary
- Because of the heterogeneity stated in Problem 3, industry statistics are not very useful.
- The reports the reinsurer receives may be lacking some important information.
- Because of the heterogeneity in coverage and reporting requirements, reinsurers often have data coding and IT systems problems.
- The size of an adequate loss reserve compared to surplus is greater for a reinsurer.

(僅須回答其中六項)

29.（2 分）

依據 Venter, "Tails of Copulas"，請定義何謂 copulas。

【參考解答】

Copulas provide a convenient way to express joint distributions of two or more random variables. A copula separates the joint distribution into two contributions: the marginal distributions of the individual variables, and the interdependency of the probabilities. One basic result is that any joint distribution can be expressed in this manner. Another convenience is that the conditional distributions can be readily expressed from the copula.

30.（2 分）

依據 Goldfarb, "P&C Insurance Company Valuation"，針對公司價值評估之折現現金流量法(Discount Cash Flow)，請回答下列問題：

(1)請簡述折現現金流量法的二種不同評估方式。(1 分)

(2)請說明二種評估方式採用折現率的主要差異為何。(1分)

【參考解答】

(1) Free Cash Flow to the Firm (FCFF) approach:
FCFF represents the cash that could be paid to all sources of capital, including both the debtholders and the equity holders. Discounting the FCFF produces a value for the entire firm. The value of the equity portion of the firm is then determined by subtracting the market value of the debt from the total firm value.
Free Cash Flow to Equity (FCFE) approach:
FCFE represents the cash generated by the firm which could be paid to the shareholders of the firm. The resulting valuation thus represents the equity valuation directly by determining the present value of these free cash flows.

(2) The FCFF approach uses a discount rate that reflects the overall risk to both debtholders and equity holders (a so-called weighted average cost of capital);
the FCFE approach uses a discount rate that reflects the risk to the equity holders only.