# G6 高等費率釐定

1.(6 分)

給定下列保險人的承保資訊：

| 保單 | 保費 | 保單限額 | 損失金額 |
|---|---|---|---|
| 1 | $12,000 | $200,000 | $150,000 |
| 2 | 3,000 | 50,000 | 50,000 |
| 3 | 30,000 | 500,000 | 300,000 |

(a) 假設這些保單利用固定 60%比例再保險. (2 分)

i. 計算原保險人的自留保費.

ii. 計算原保險人的自留損失.

(b) 假設這些保單利用溢額合約再保險，原保險人自留 $100,000 一線的風險，再保合約共包含九線與最大上限為 $900,000. (2 分)

i. 計算原保險人的自留保費.

ii. 計算原保險人的自留損失.

(c) 給定何種理由，原保險人會購買溢額合約再保險，而不是固定比例再保險. (1 分)

(d) 給定何種理由，原保險人會購買固定比例再保險，而不是溢額合約再保險. (1 分)

【參考解答】

2007 Exam 6- Q27 (2.5 points)

(a) A 60% quota share means the primary company retains 40% of the premium and losses.

i. Retained premium = 40% × ($12,000 + $3,000 + $30,000) = $18,000

ii. Retained losses = 40% × ($150,000 + $50,000 + $300,000) = $200,000

(b) First we need to determine the % retained for each policy.

| (1) policy | (2)limit | (3) max[0%,min[$900,000, (2)-$100,000]/(2)] | (4)100-(3) |
|---|---|---|---|
| 1 | $200,000 | 50% | 50% |
| 2 | $50,000 | 0 | 100% |
| 3 | %500,000 | 80% | 20% |

i. Retained premium = (50%)($12,000) + (100%)($3,000) + (20%)($30,000) = $15,000

ii. Retained losses = (50%)($150,000) + (100%)($50,000) + (20%)($300,000) = $185,000

(c) Surplus share would allow the primary company to retain more of the lower limit risks while ceding a higher percentage of losses for higher limits.

(d) Quota share policies are much simpler to manage since surplus share require keeping track of the percent ceded for each policy.

2.(3 分)

某精算人員採用logistic model 評估保戶續保率資料，發現有2個顯著影響續保率之變數，分別為費率調幅及客服人員過去一年打電話給保戶的次數，相關參數估計如下:

| 費率調幅 | 參數估計 |
|---|---|
| <3% | 0.35 |
| >=3%&<8% | 0 |
| >=8% | -0.45 |

| 客服人員打電話次數 | 參數估計 |
|---|---|
| 0 | 0 |
| 1 | -0.22 |
| >=2 | -0.43 |

(1)假設某客戶今年費率調幅=8%且公司客服人員過去一年打過 2 次電話給保戶，請計算該客戶之續保率。(1.5 分)

(2)假設公司希望續保率能維持在 80%以上，可能採用的經營策略是增加客服人員打電話給保戶的次數作為費率結構中風險分類因子。請說明你贊成或反對此策略並提供 2 個說明理由。(1.5 分)

【参考解答】

(1)

$$E(Y_i) = g^{-1}(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4)$$

For a logistic model, a logit link function is used:

$$g^{-1}(x) = \frac{e^x}{1 + e^x}$$

For a customer with a 8% rate increase and 2 phone calls:

$$E(Y_i) = g^{-1}(0 + 0.35 * 0 + (-0.45) * 1 + (-0.22) * 0 + (-0.43) * 1)$$

$$= g^{-1}(-0.88)$$

$$= \frac{e^{-0.88}}{1 + e^{-0.88}} = 0.293$$

So the predicted renewal probability for this customer is 29.3%

(2)

I would be against adding the number of phone calls to the classification plan because:

i. It can be easily manipulated by the insured

ii. The variable lacks constancy in that the number of phone calls for an insured might change dramatically from year to year.

3.(6 分)

某精算人員針對 excess of loss workers compensation policy 進行定價，資料如下:

- injury types: fatal, permanent total injury (PT), major permanent partial (Major), minor permanent partial (Minor), temporary total (TT), and medical-only (Med).
- A multi-dimensional credibility technique (predicted) was used to estimate the frequency for class 5160.
- Class 5160 is in hazard group F.
- The hazard group relativities for Major, Minor, TT, and Med will be used.
- The multi-dimensional credibility relativities for PT claims will be used.
- Class 5160 is in Quintile 4 for both Fatal and PT claims.

| Hazard Group F | | | | | | |
|---|---|---|---|---|---|---|
| | Fatal | PT | Major | Minor | TT | Med |
| Frequency Relativity to TT | 0.005 | 0.005 | 0.08 | 0.35 | 1.00 | 3.2 |
| Severity Relativity to TT | 90 | 120 | 40 | 5 | 1.00 | 0.4 |
| Loss Elimination Ratio at $250,000 | 30% | 25% | 60% | 100% | 100% | 100% |

| | |
|---|---|
| TT Frequency per $100 payroll | 0.00025 |
| TT Severity for Hazard Group F | $10,000 |

| Hazard Group F for Fatal Claims | | | |
|---|---|---|---|
| | Predicted | Raw Data | Holdout Sample |
| Quintile 1 | 0.85 | 0.75 | 0.92 |
| Quintile 2 | 0.95 | 0.90 | 0.96 |
| Quintile 3 | 1.00 | 1.00 | 1.00 |
| Quintile 4 | 1.05 | 1.10 | 1.04 |
| Quintile 5 | 1.15 | 1.25 | 1.08 |
| Mean | 1.00 | 1.00 | 1.00 |

| Hazard Group F for PT Claims | | | |
|---|---|---|---|
| | Predicted | Raw Data | Holdout Sample |
| Quintile 1 | 0.75 | 0.80 | 0.92 |
| Quintile 2 | 0.85 | 0.90 | 0.96 |
| Quintile 3 | 1.00 | 1.00 | 1.00 |
| Quintile 4 | 1.15 | 1.10 | 1.04 |
| Quintile 5 | 1.25 | 1.20 | 1.08 |
| Mean | 1.00 | 1.00 | 1.00 |

(1)請評估是否應採用multi-dimensional credibility relativities 計算 expected loss for fatal claims。(2分)

(2)依據第(1)小題結果，請計算expected loss for an excess of $250,000 workers compensation policy with $10 million in payroll。(4分)

【参考解答】

(1)
We want to use either the Hazard Group, credibility technique, or the raw data for fatal claims, based on
which has the lowest SSE (compared to the holdout sample).
SSEPredicted = (0.85 - 0.92)^2 + (0.95 - 0.96) ^2 + (1.00 - 1.00) ^2 + (1.05 - 1.04) ^2 + (1.15 - 1.08) ^2 = 0.05
SSEHazard = (1.00 - 0.92) ^2 + (1.00 - 0.96) ^2 + (1.00 - 1.00) ^2 + (1.00 - 1.04) ^2 + (1.00 - 1.08) ^2 = 0.025
SSERaw = (0.75 - 0.92) ^2 + (0.90 - 0.96) ^2 + (1.00 - 1.00) ^2 + (1.10 - 1.04) ^2 + (1.25 - 1.08) ^2 = 0.085
The Predicted relativities result in the lowest Sum of Squared Errors for fatal claims, so they should be used for multi-dimensional credibility relativities instead of the fatal claims or the raw data relativities.

(2)
I assume that the payroll is all for class 5160.
For Med, TT, and Minor, the contribution will be $0 since the Loss Elimination Ratios are 100%,
Major: [($10,000,000 / $100)*0.00025*0.08*1.00]* [$10,000*40]*(1 - 60%) = $320,000
Since PT  and Fatal uses the multi-dimensional credibility relativities, look up the value for predicted for Quintile 4
for PT claims to get a relativity of 1.15 relative to the hazard group frequency.
PT: [($10,000,000 / $100)*0.00025*0.005*1.15]* [$10,000*120]*(1 - 25%) = $129,375
for Fatal claims to get a relativity of 1.05 relative to the hazard group frequency.
Fatal: [($10,000,000 / $100)*0.00025*0.005*1.05]* [$10,000*90]*(1 - 30%) = $82,688
Total Expected Excess Loss = $0 + $0 + $0 + $320,000 + $129,375 + $82,688 = $532,063

4.（5 分）
A 保險公司車險資料如下表:

| 地區 | 無肇事年度 | 2 年以上無肇事現行滿期保費 | 滿期車年 | 賠案件數 | 已發生損失 |
|---|---|---|---|---|---|
| 北部 | 0 | $10,000,000 | 10,000 | 5,000 | $8,000,000 |
| 北部 | 1 | $145,000,000 | 14,500 | 45,000 | $85,000,000 |
| 北部 | 2+ | $250,000,000 | 25,000 | 80,000 | $150,000,000 |
| 中部 | 0 | $20,000,000 | 25,000 | 6,000 | $20,000,000 |
| 中部 | 1 | $320,000,000 | 300,000 | 80,000 | $180,000,000 |
| 中部 | 2+ | $600,000,000 | 580,000 | 160,000 | $364,000,000 |
| 南部 | 0 | $10,000,000 | 10,000 | 4,000 | $8,000,000 |
| 南部 | 1 | $100,000,000 | 125,000 | 45,000 | $48,000,000 |
| 南部 | 2+ | $200,000,000 | 200,000 | 80,000 | $130,000,000 |

請選擇計算可信度時適當之 exposure base，請詳細說明選擇的適當性。

【參考解答】
Premium should be used as the exposure base to prevent the maldistribution of premium if higher frequency territories have higher premiums and territory relativities are proper. Testing this with the data shows:

| 地區 | Frequency (∑claims/∑ car years | avg Prem (∑ Prem/∑car years) | Loss Ratio |
|---|---|---|---|
| 北部 | 2.626 | 8182 | 0.6 |
| 中部 | 0.272 | 1039 | 0.6 |
| 南部 | 0.385 | 925 | 0.6 |

All territories have the same Loss Ratio, which suggests the territory relativities are proper.
However, higher frequency territories do not have higher average premiums. Therefore, it is advisable to use earned car years as the exposure base instead of earned premium.

5.(4 分)

某精算人員以下列公式估計某風險之期望損失

E = ZX + (1-Z)P, where

X = the most recent accident year’s losses

P = the prior estimate of the most recent accident year

Z = the credibility assigned to the most recent accident year

假設資料無延遲現象且Z = 10%

(1)請計算 2011 年估計值及 2015 年估計值中 2010 年損失的權重之差異值。(2 分)

(2)假設風險參數發生險種改變(significant shifts in risk parameters)造成 Z 需要重新評估，且其他因素皆相同。請問第(1)小題的答案會增加、減少或者不變，請詳細說明支持你的答案之理由。(2分)

【參考解答】

(1)

Note that E for each year becomes the value of P for the following year.

E02 = 0.1X01 + 0.9P01

E03 = 0.1X02 + 0.9[0.1X01 + 0.9P01]

E04 = 0.1X03 + 0.9E03

E05 = 0.1X04 + 0.9E04 = 0.1X04 + _ _ _ + (0.1)(0.9)3X01

(0.1)(0.9)^3 = 0.0729

The weight given to AY 2010 losses in the 2011 estimate is the coefficient of X01 in the formula for E02, which is 10%. The weight given to AY 2010 losses in the 2015 is the coefficient of X01 in the formula for E05.

Difference= 0.1 -0.0729 = 0.0271

(2)If there is a significant shift in risk parameters, then historical data will be less predictive of future data. As such, relatively more weight should be given to recent historical data and less weight should be given to older historical data. This would cause Z to increase. With Z being larger, the value of Z-(Z)(1- Z)^3 will also increase.

6.（4 分）
請依據Robertson’ s ” NCCI’ s 2007 Hazard Group Mapping”一文中採用之proposed hazard groups ，詳細說明3項符合American Academy of Actuaries’ ” Risk Classification Statement of Principles.”之考量。

【參考解答】
New hazard groups reflect the 3 statistical considerations of the AAA

1.Credibility: assigning a credibility of $z = \min(\frac{n}{n+k} * 1.5, 1)$gives larger classes more weight to permit the calculation of more accurate predictors of excess ratios.
2.Homogeneity: clustering analysis using k-means algorithm is used to assign each class into a hazard group. k-means has the property to minimize within variance and maximize between variance so new hazard groups are homogeneous.
3.Predictive Stability: since current hazard groups were used as the complement of credibility, this provided stability in the class excess ratios, while still recognizing any changes that were credible.

7.（3 分）

給定下列回溯保費機制的資訊：

| | |
|---|---|
| 預期總損失 | $120,000 |
| 最大損失 | $240,000 |
| 最小損失 | $60,000 |
| 費用與利潤邊際(排除稅) | $24,000 |
| 損失轉換因子 | 1.35 |

下列為同等大小損失的 Table M：

| Entry Ratio | Charge |
|---|---|
| 0.25 | 0.85 |
| 0.50 | 0.76 |
| 0.75 | 0.69 |
| 1.00 | 0.64 |
| 1.25 | 0.59 |
| 1.50 | 0.56 |
| 1.75 | 0.52 |
| 2.00 | 0.49 |
| 2.25 | 0.46 |
| 2.50 | 0.44 |
| 2.75 | 0.42 |

請計算基礎保費(Basic premium)

【參考解答】

E=120,000

G-240,000

H=60,000

e=24,000

C=1.35

b=e-E(c-1)+cI=24,000-120,000x.35+1.35x(120,000(Φ(rG)- Ψ(rH))

rG=LossG/E=240,000/120,000=2　Φ(2)=0.9

rH=LoosH/E=60,000/120,000=0.5　Ψ(0.5)=0.49

Ψ(0.5)= Φ(0.5)+0.5-1=0.76+0.5-1=0.26

b=24,000-120,000x.35+1.35x120,000x(0.49-0.26)=19,260

8.（3 分）

依據 Gillam and Snader, “Fundamentals of Individual Risk Rating”及下列資訊，請計算經驗調整係數(experience modification)。

- 預期賠款：$3,500,000
- 預期超額賠款：$1,300,000
- 基層可信度：0.65
- 超額可信度：0.35
- 實際賠款：$7,000,000
- 實際超額賠款：$2,000,000

【參考解答】

$M = (Z_pA_p+(1- Z_p)E_p + Z_eA_e+(1- Z_e)E_e) / E$

$= (0.65\times(7000-2000)+0.35\times(3500-1300)+0.35\times2000+0.65\times1300)/3500$

$= 1.59$

9.（3 分）

依據 Gillam, “Workers’ Compensation Experience Rating: What Every Actuary Should Know”，

(1) 請定義何謂經驗費率計畫之平衡調整(off-balance)。（1.5 分）

(2) 若損失成本升高致費率適足性持續惡化，請問平衡調整會如何影響所需保費水準之計算？（1.5 分）

【參考解答】

(1) Experience rating is a term used to mean standard premium divided by manual premium or, put another way, manual premium weighted average modification.

(2) When rates are inadequate, the off-balance will tend to be greater than unity, moderating the extent of inadequacy. Since rate calculations are based on standard, not manual, premium, this debit off-balance will produce a lower indicated loss ratio and a smaller indicated rate level change.

10.（3 分）

依據下列五個相似的風險的損失經驗範例：

| 風險 | 年度損失 |
|---|---|
| 1 | $4,200,000 |
| 2 | $2,800,000 |
| 3 | $1,600,000 |
| 4 | $X |
| 5 | $Y |

平均數$2,000,000

請計算 insurance saving Ψ(0.8)

【參考解答】

($4,200,000 + $2,800,000 + $1,600,000 + X + Y) / 5 = $2,000,000

解出 X + Y：X + Y = $1,400,000

entry ratio 為 0.8 的損失為 = 0.8×$2,000,000 = $1,600,000.

X 與 Y 將會是唯二低於$1,600,000 的損失

(因 X 與 Y 大於 0，且 X + Y = $1,400,000)

使用垂直法(單位為仟元)，帶入(X + Y):

$$\Psi(0.8)E[A] = \frac{1}{5}(\$1,600 - X) + \frac{1}{5}(\$1,600 - Y) = \$640 - \frac{1}{5}(X + Y) = \$640 - \frac{1}{5}(\$1,400) = \$360$$

$$\Psi(0.8) = \$360/\$2,000 = 0.18$$

11.(4 分)

依據 Gilliam and Snadar 以及下列全保障保單(full-coverage policies)資訊：

自負額$25,000 時的 LER 20%

預期損失率(含 ALAE) 60%

取得費用預備(Provision) 15% (Provision of acquisition expense)

稅的預備 3% (Provision of Tax)

預期利潤 5%

檢查費用預備(Provision) 2% (Provision of inspection expense)

ULAE 的預備(Provision) 5% (Provision of ULAE)

經營費用預備(Provision) 10% (Provision of home office expense)

全保障保費 $1,000 (full-coverage premium)

請計算自我保險自留額計劃中超額保障的指示的折扣(indicated discount for excess coverage over a self-insurance retention)

- ALAE 已包含於損失中 ALAE is eliminated in proportion to losses
- 取得費用、稅與利潤隨保費變動而變動 Acquisition, taxes and profit vary with premium
- 檢查費用、ULAE 與 50%的經營費用隨賠款變動而變動 Inspection, ULAE and 50% of home office expense vary with losses
- 其它的費用為全保障保費的 3%

【參考解答】

$$D=\frac{(\mathrm{fkE})(1+i_E+u_E+gh_E)}{1-A-T-P}=\frac{(1)(0.2)(0.6)(1+0.02+0.05+(0.5)(0.1))}{1-0.15-0.03-0.05}=0.175$$

indicated discount= full-coverage premium×D=1,000*0.175=175

12. (4 分)

進行費率係數分析時，常建議以 GLM 等多維度分析取代傳統單維度分析(one-way analysis)。

請簡述說明採傳統單維度分析計算費率係數的二個主要缺點，並舉例說明之。

【參考解答】

- One-way analyses can be distorted by correlations between rating factors. For example, young drivers may in general drive older cars. A one-way analysis of age of car may show high claims experience for older cars, however this may result mainly from the fact that such older cars are in general driven more by high risk younger drivers.
- One-way analyses also do not consider interdependencies between factors in the way they affect claims experience. These interdependencies, or interactions, exist when the effect of one factor varies depending on the levels of another factor. For example, the pure premium differential between men and women may differ by levels of age.

13.(5 分)

某保險公司的一般責任保險，損失頻率為期望值為 0.1 波阿松分配(Poisson frequency distribution)，損失幅度為 lognormal 分配，且其第一動差與第二動差如下表：

| 每事故限額 | 每事故限額下的損失幅度 | |
|---|---|---|
| | 第一動差 | 第二動差 |
| \$100,000 | 38,866 | $2.5680\times10^{9}$ |
| \$500,000 | 65,846 | $1.4592\times10^{10}$ |
| \$1,000,000 | 76,128 | $3.0620\times10^{10}$ |
| \$2,000,000 | 84,488 | $5.4400\times10^{10}$ |
| \$5,000,000 | 92,218 | $1.0318\times10^{11}$ |
| \$10,000,000 | 95,802 | $1.5348\times10^{11}$ |

假設：

- 高保額係數只反映了過程風險
- 每事故限額\$100,000 下，合適的風險加成為 5%的期望純保費 The appropriate risk load for policies having a \$100,000 per occurrence limit is equal to 5% of the expected pure premium.
- 每事故基礎限額為\$100,000 The basic limit is \$100,000 per occurrence

請計算每事故限額\$5,000,000 下，風險調整後的高保額係數 Calculate the risk-adjusted increased limit factor for a policy having a \$5,000,000 per occurrence limit.

【參考解答】

答案一

λ = (0.05 ×38,866) / (2.5680×$10^9$) = 0.000000757

每事故限額$5,000,000下，風險調整後的高保額係數

$\frac{92,218 + 0.000000757\times 1.0318\times10^{11}}{38,866 + 0.000000757\times 2.5680\times10^{9}}$=4.17

答案二

a) E(s) = E(n) × E(x)

Var(s) = E(n) × Var(x) + Var(n) ×$[E(x)]^2$

期望值為Poisson分配，故E(n) = Var(n)

Var(s) = E(n) × [Var(x) +$E(x)^2$]= E(n) × [E($x^2$) －$E(x)^2$ + $E(x)^2$]= E(n) ×E($x^2$)

風險加成 = λ × Var(s)

基礎限額:

E(PP) = E(n) × E(x)= 0.10 × (38,866)= 38,86.60

Var (PP) = 0.10 × (2.5680×$10^9$)= 2.5680×$10^8$

風險加成= 0.05 ×38,86.60 = λ ×2.5680 ×$10^8$

求 λ, λ = 7.567 ×$10^{-7}$

ILF5M = [E5M(PP) + λ × Var5M(PP)]

[EBL(PP) + λ × VarBL(PP)]= (0.10 ×92,218) + (7.567 ×$10^{-7}$ × 0.10 ×$1.0318\times10^{11}$)×(0.10 ×38,866) + (7.567 ×$10^{-7}$× 0.10 ×2.5680×$10^9$)= 4.17

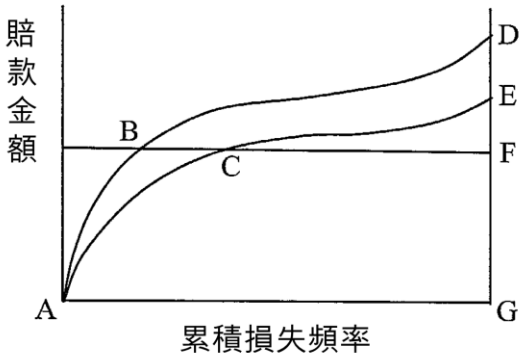
14.(5 分)

經分析得知某責任險所有賠款通貨膨脹率為 5%，依據 Lee, “The Mathematics of Excess of Loss Coverages and Retrospective Rating－A Graphical Approach”，

(1) 請圖形表示基本限額(basic limits)通貨膨脹率及高層(excess limits)通貨膨脹率，並與整體通貨膨脹率 5%進行比較。(2.5 分)

(2) 針對高層通貨膨脹率與整體通貨膨脹率 5%的比較結果，請敘述二項主要原因為何。(2.5 分)

【參考解答】

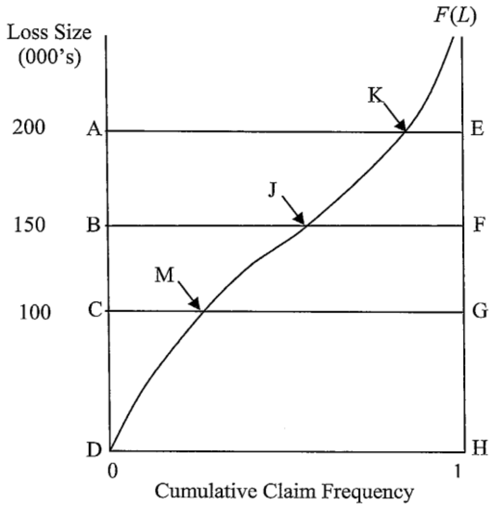



(1) 基本限額通貨膨脹率 ＝ABFG / ACFG – 1 < 5%
高層通貨膨脹率 ＝BDF / CEF – 1 > 5%

(2) For losses that are between the basic limits divided by 1.05 and the basic limit, the increase is split between the basic limits and excess portions, and previous they had no excess portion.
For loss equal to or greater than the basic limit, the entire impact of inflation is applied to the excess portion.

15.（7 分）

下圖繪出了基礎限額$100,000 時的累積損失幅度機率分配：




a. （3 分）

請運用上圖各端點的英文字母表達出限額$150,000 的高保額係數。

b. （3 分）

令 I(L)為限額 L 的高保額係數，請運用上圖各端點的英文字母表達出限額$150,000 的的微分。

c. （1 分）

請問 Lee 在 " The Mathematics of Excess of Loss Coverages and Retrospective rating - A Graphical Approach" 一文中是如何運用上圖證明 Miccolis 的一致性測試？

【參考解答】

a. $I(150)=\frac{E[g(X;150)]}{E[g(X;100)]}=\frac{JFHD}{MGHD}$

b. $I'(150)=\frac{G(150)}{E[g(X;100)]}=\frac{JF}{MGHD}$

c. Lee divided the region below the curve into horizontal layers of equal width. It is evident that the expected payment in a higher layer must be less or equal to that in a lower layer because the curve bends upward and to the right. ILFs must increase at a decreasing rate as the limit increases.

用水平線將上圖切出不同的保額層，會發現較高保額層的期望損失一定會小於或等於較低保額層的期望損失，另外累積損失幅度機率分配曲線是向上彎曲的，代表高保額係數一定是以邊際遞減的方式隨保額增加而增加。

16.(6 分)

某位精算師正在評比不同的經驗費率計劃，請看以下資訊：

每位保戶都有高於$100,000的保費

| 保戶的Experience Mods | 損失率標準保費下之損失率 | | | |
|---|---|---|---|---|
| | A | B | C | D |
| 0.800-0.899 | 0.985 | 1.020 | 0.999 | 0.995 |
| 0.900-0.999 | 0.989 | 1.015 | 1.005 | 0.999 |
| 1.000-1.099 | 1.022 | 1.000 | 1.012 | 1.000 |
| 1.100-1.199 | 1.015 | 0.985 | 0.995 | 1.005 |
| 1.200-1.299 | 1.025 | 0.980 | 1.015 | 1.015 |

(a) (1.5分)

請問哪個計劃表現最佳，請說明

(b) (4.5分)

請問除了最佳計劃，其它三個計劃中哪個計劃表現最佳，請說明

【參考解答】

a. 計劃 C 表現最佳，因為損失率都很接近 1 並且損失率並未隨著 Mods 的增加而有明顯的趨勢

b. 計劃 A 是剩餘的計劃中表現最佳的，因為計劃 A 比 C 略為分散，他的損失率雖有略為增加，但是尚未表現出加費的保戶或減費的保戶在加減費後損失率仍有顯著的區別，計劃 B 也比 C 略為分散，但他下降的損失率表現出加費的保戶或減費的保戶在加減費後損失率仍有顯著的區別，計劃 D 分散的情況跟 C 差不多，但是上升的損失率表現出加費的保戶或減費的保戶在加減費後損失率仍有顯著的區別。

17. (10 分)

依據下列資訊，請分別計算 Table M Charges and Savings 及 Table L Charges and Savings，損失率請由 0%計算至 120%，並以 10%遞增。

| 風險個數 (# of Risks) | 無限制損失率 (Unlimited loss ratio) | 風險個數 (# of Risks) | 無限制損失率 (Limited loss ratio) |
|---|---|---|---|
| 4 | 10% | 5 | 10% |
| 3 | 20% | 4 | 20% |
| 2 | 30% | 2 | 30% |
| 2 | 50% | 2 | 50% |
| 1 | 60% | 2 | 60% |
| 1 | 70% | 1 | 80% |
| 1 | 80% | 2 | 90% |
| 2 | 90% | 3 | 100% |
| 1 | 100% | | |
| 1 | 110% | | |
| 2 | 120% | | |

【參考解答】

E = (4×10%+3×20%+…+2×120%)/20 = 0.55

E[A*] = (5×10%+4×20%+…+3×100%)/20 = 0.462

K = 1-.462/.55 = .16

Table M:

| Loss ratio | Entry ratio | # of Risks at ratio | Risks/ Total risks | Risk at higher ratios | Loss at next layer | Table M Charge | Table M Saving |
|---|---|---|---|---|---|---|---|
| 0.0 | 0.000 | 0 | 0.000 | 1.000 | 0.182 | 1.000 | 0.000 |
| 0.1 | 0.182 | 4 | 0.200 | 0.800 | 0.145 | 0.818 | 0.000 |
| 0.2 | 0.364 | 3 | 0.150 | 0.650 | 0.118 | 0.673 | 0.036 |
| 0.3 | 0.545 | 2 | 0.100 | 0.550 | 0.100 | 0.555 | 0.100 |
| 0.4 | 0.727 | 0 | 0.000 | 0.550 | 0.100 | 0.455 | 0.182 |
| 0.5 | 0.909 | 2 | 0.100 | 0.450 | 0.082 | 0.355 | 0.264 |
| 0.6 | 1.091 | 1 | 0.050 | 0.400 | 0.073 | 0.273 | 0.364 |
| 0.7 | 1.273 | 1 | 0.050 | 0.350 | 0.064 | 0.200 | 0.473 |
| 0.8 | 1.455 | 1 | 0.050 | 0.300 | 0.055 | 0.136 | 0.591 |
| 0.9 | 1.636 | 2 | 0.100 | 0.200 | 0.036 | 0.082 | 0.718 |
| 1.0 | 1.818 | 1 | 0.050 | 0.150 | 0.027 | 0.045 | 0.864 |
| 1.1 | 2.000 | 1 | 0.050 | 0.100 | 0.018 | 0.018 | 1.018 |
| 1.2 | 2.182 | 2 | 0.100 | 0.000 | 0.000 | 0.000 | 1.182 |

Table L:

| Loss ratio | Entry ratio | # of Risks at ratio | Risks/ Total risks | Risk at higher ratios | Loss at next layer | Table M Charge | Table M Saving |
|---|---|---|---|---|---|---|---|
| 0.0 | 0.000 | 0 | 0.000 | 1.000 | 0.182 | 1.000 | 0.000 |
| 0.1 | 0.182 | 5 | 0.238 | 0.762 | 0.139 | 0.818 | 0.000 |
| 0.2 | 0.364 | 4 | 0.190 | 0.571 | 0.104 | 0.680 | 0.043 |
| 0.3 | 0.545 | 2 | 0.095 | 0.476 | 0.087 | 0.576 | 0.121 |
| 0.4 | 0.727 | 0 | 0.000 | 0.476 | 0.087 | 0.489 | 0.216 |
| 0.5 | 0.909 | 2 | 0.095 | 0.381 | 0.069 | 0.403 | 0.312 |
| 0.6 | 1.091 | 2 | 0.095 | 0.286 | 0.052 | 0.333 | 0.424 |
| 0.7 | 1.273 | 0 | 0.000 | 0.286 | 0.052 | 0.281 | 0.554 |
| 0.8 | 1.455 | 1 | 0.048 | 0.238 | 0.043 | 0.229 | 0.684 |
| 0.9 | 1.636 | 2 | 0.095 | 0.143 | 0.026 | 0.186 | 0.823 |
| 1.0 | 1.818 | 3 | 0.143 | 0.000 | 0.000 | 0.160 | 0.978 |
| 1.1 | 2.000 | 0 | 0.000 | 0.000 | 0.000 | 0.160 | 1.160 |
| 1.2 | 2.182 | 0 | 0.000 | 0.000 | 0.000 | 0.160 | 1.342 |

18.(6 分)

**下列表格代表再保險人對於單一原保險人契約的累積損失分佈：**

| 損失率區間範圍 | 區間平均損失 | 區間損失機率 |
|---|---|---|
| 0-60% | 47.9% | 0.45 |
| 60-75% | 67.8% | 0.29 |
| 75-90% | 81.5% | 0.17 |
| 90%以上 | 99.6% | 0.09 |

**原保險人對於 60%至 75%的損失將承受 50%，對於 75%至 90%的損失將承受 80%，試計算再保險人經過損失調整後的預期損失率。**

【參考解答】

| 損失率區間範圍 | 區間平均損失 | 區間損失機率 | 調整後 |
|---|---|---|---|
| 0% to 60% | 47.9% | 0.45 | 47.9% |
| 60% to 75% | 67.8% | 0.29 | |
| | | 63.9% = 60% + 50%(67.8%-60%) | |
| 75% to 90% | 81.5% | 0.17 | |
| | 68.8% = 60% + 50%(75%-60%) + 20%(81.5%-75%) | | |
| 90% or above | 99.6% | 0.09 | |
| 80.1% = 60% + 50%(75%-60%) + 20%(90%-75%) + (99.6%-90%) | | | |

預期損失率 Expected LR net of corridor

= 0.45(47.9%) + 0.29(63.9%) + 0.17(68.8%) + 0.09(80.1%) = 59.0%

19. (8 分)

給定下列一年期回溯費率計畫(Retrospective rating plan)相關資訊，請計算其最高保費比率(Maximum premium ratio)。

- 標準保費(Standard premium)：$3,000,000
- 最低保費比率(Minimum premium ratio)：0.66
- 基本保費比率(Basic premium ratio)：0.25
- 預期損失率(Expected loss ratio)：0.60
- 賠款轉換係數(Loss conversion factor)：1.1
- 費用率(Expense provision)：0.24
- 稅負乘數(Tax multiplier)：1.1
- Table M Charges：

| | Expected losses (萬元) | | |
|---|---|---|---|
| Entry ratio | 50-100 | 100-150 | 150-200 |
| 0.4 | 0.72 | 0.67 | 0.62 |
| 0.5 | 0.6 | 0.56 | 0.52 |
| 0.6 | 0.49 | 0.46 | 0.43 |
| 0.7 | 0.4 | 0.37 | 0.35 |
| 0.8 | 0.32 | 0.29 | 0.27 |
| 0.9 | 0.24 | 0.21 | 0.19 |
| 1.0 | 0.16 | 0.13 | 0.12 |
| 1.1 | 0.13 | 0.11 | 0.09 |
| 1.2 | 0.09 | 0.08 | 0.07 |

【參考解答】

$H = (b + cL_H) \cdot T = (b + cr_H E) \cdot T$　　$r_H = (.66/1.1-.25)/(1.1\times.6) = .530$

Expected losses = 3000000×.6 = 1800000

$X_H = .52+(.53-.5)(.43-.52)/(.6-.5) = .493$

$S_H = X_H + r_H - 1 = .493+.53-1 = .023$

$b-e+(c-1)E = c\,E\,(X_G - S_H)$　　$X_G = .023+(.25-.24+(1.1-1).6)/(1.1\times.6) = .129$

$r_G = .9+(.19-.129)(1-.9)/(.19-.12) = .987$

$G = (b + cr_G E) \cdot T = (.25+1.1\times.987\times.6) \cdot 1.1 = .992$

20.(5 分)

某精算人員採用 Generalized Linear Model 進行費率釐訂，並取得下列損失頻率(單位:每 1000 車年)資料

| | 小客車 | 小貨車 | 大貨車 |
|---|---|---|---|
| 北區 | 12 | 15 | 25 |
| 南區 | 8 | 12 | 22 |

(1)請列出 Generalized Linear Model 模型之 design matrix 及 vector of beta parameters。(2 分)

(2)請分別針對 Poisson 及 gamma error structures, 說明 relationship between the variance and the expected value and how these relationships differ. (2 分)

(3)當 link function and error structure 選定,請敘述決定最終 beta parameters 的過程. (1 分)

【參考解答】

(1)

Let $X_1$ = 1 if 北區

Let $X_2$ = 1 if 南區

Let $X_3$ = 1 if 小客車

Let $X_4$ = 1 if 小貨車

Let $X_5$ = 1 if 大貨車

| | 小客車 | 小貨車 | 大貨車 |
|---|---|---|---|
| 北區 | Y1 | Y2 | Y3 |
| 南區 | Y4 | Y5 | Y6 |

To prevent intrinsic aliasing, I will remove the 南區 and 大貨車 levels and add in an intercept term:

$$\text{design matrix } X = \begin{matrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{matrix}$$

beta parameters =[$\beta_{intercept}$ , $\beta_{北區}$, $\beta_{小客車}$, $\beta_{小貨車}$]

(2)

For Poisson, variance = expected value

For Gamma, variance = (expected value)^2

(3)

i. Identify the likelihood function

ii. Take the log to turn the product of several items into a sum

iii. Maximize the log of the likelihood function and set the partial derivatives for each parameter to zero

iv. Solve the resulting system of equations

v. Compute the predicted values