G5 高等準備金，價值評估與企業風險管理

計算及申論題共計 26 題 ：

1.（5 分）
A保險公司損失資料如下:

	Cumulative Loss Payments		
Accident Year	12 Months	24 Months	36 Months
2014	2,000	3,200	4,000
2015	2,200	3,300	
2016	2,400		

假設所有年度暴露數與保費皆相同且36個月後無損失發展。

（1）請分別採用 loss-ratio based payout factors 及 Benktander method 計算 2015 年底之準備金金額。(1 分)
（2）請針對 AY 2016 年計算 Benktander method 第五次疊代之準備金金額。(2 分)
（3）假設$\text{Var}(U_i) = \text{Var}(U_i^{BC})$，請採用 Hurlmann Method 在最佳可信度及最小變異數前提下，估計 AY 2016 年未付賠款金額。(2 分)

【參考解答】
(1)
Avg. Paid:
0-12 months = (2,000 + 2,200 + 2,400)/3 = 2,200
12-24 months = (1,200 + 1,100)/2 = 1,150
24-36 months = 800
U0 = 2,200 + 1,150 + 800 = 4,150
P1 = 2,200/4,150 = .53; Q1 = 1 - .53 = .47
P2 = (2,200 + 1,150)/4,150 = .807; Q1 = 1 - .807 = .193

1st iteration ultimate losses
2015 = 4,150 × .193 + 3,300 = 4,100
2016 = 4,150 × .47 + 2,400 = 4,350

2nd iteration ultimate losses
2015 = 4,100 × .193 + 3,300 = 4,090
2016 = 4,350 × .47 + 2,400 = 4,444

Total estimated Benktander outstanding losses as of December 31, 2016 =
4,090 + 4,444 – 3,300 – 2,400 = 2,834

(2)
3rd iteration ultimate losses from part A 2016 = 4,444 × .47 + 2,400 = 4,488
4th iteration ultimate losses from part A 2016 = 4,488 × .47 + 2,400 = 4,509
5th iteration ultimate losses from part A 2016 = 4,509 × .47 + 2,400 = 4,519
Reserve = 5th iteration Ultimate minus paid = 4,519 – 2,400 = 2,119

(3)
$Z^* = p1 / (p1 + \sqrt{p1}) = 0.53 / (0.53 + \sqrt{0.53}) = 0.421$
RC = Z* × Rind + (1 – Z*) × Rcoll = 0.421 ×4,527+ (1 – 0.421) ×4,350 = 4,425
Reserve = Estimated Ultimate minus Paid = 4,425 – 2,400 = 2,025

2.（4 分）

依據下列 2016 年底之資料:

Accident Year	On-level Premiums	Cumulative Paid Loss	Fitted Paid Emergence Pattern
2013	$300,000	$120,000	70%
2014	$250,000	$80,000	45%
2015	$320,000	$50,000	25%
2016	$280,000	$20,000	15%

Cape Cod Method

Parameter standard deviation = 150,000

Process variance/mean scale parameter (σ^2)= 2,500

LDF Method

Parameter standard deviation = 250,000

Process variance/mean scale parameter (σ^2)= 3,000

（1）請計算 total standard deviation of the total loss reserve indication resulting from the Cape Cod Method。(1 分)

（2）請計算 total standard deviation of the total loss reserve indication resulting from the LDF Method。(1 分)

（3）請說明 LDF Method 的σ^2大於 Cape Cod Method 的σ^2的原因。(2 分)

【參考解答】

(1)

Used up premium = 300k*0.7+250k*0.45+320k*0.25+280k*0.15=444.5K

ELR=(120+80+50+20)/444.5=0.607

CC RESERVE=[300k*(1-0.7)+250k*(1-0.45)+320k*(1-0.25)+280k*(1-0.15)]*0.607

=428,538

Process Standard Deviation =(2500*428538)^0.5=32,731

Total Standard Deviation =(32731^2+150000^2)^0.5=153,530

(2)

LDF RESERVE=120k*(1/0.7-2)+80k*(1/0.45-1)+50k*(1/0.25-1)+20k*(1/0.15-1)

=412,540

Process Standard Deviation =(3000*412540)^0.5=35180

Total Standard Deviation =(35180^2+250000^2)^0.5=252463

(3)

Since LDF uses more parameters than Cape Cod, it has a higher σ^2 since σ^2 penalizes for using too many parameters (by dividing by (n-p))

3.（4 分）

依據下列 2016 年底之資料:

Accident Year	Cumulative Reported Loss @ 24 Months	Ultimate Loss
2013	$30,000	$70,000
2014	$35,000	$75,000
2015	$30,000	$65,000
2016	$26,000	

（1）請使用 least squares method 計算 AY 2016 之最終損失金額。（1 分）

（2）請針對下列情境，舉出並說明一個採用 least squares method 面臨的問題。(1 分)

a. The slope parameter is negative.

b. The intercept parameter is negative.

（3）請考量下列情境之改變。(2 分)

a. No change in the reporting pattern.

b. Standard deviation of reported loss as of 24 months will be 8% of estimates ultimate loss.

c. Expected ultimate loss for AY 2016 will decrease 30%.

d. Standard deviation of AY 2016 ultimate loss is expected to be $9,000.

請使用 Bayesian credibility method 計算 AY 2016 之最終損失金額。

【參考解答】

(1)

$\bar{X}$=(30+35+30)/3=31.67

$\bar{Y}$=(70+75+65)/3=70

$\overline{XY}$=(30*70+35*75+30*65)/3=2225

$\overline{X^2}$=(30^2+35^2+30^2)/3=1008

b=(2225-31.67*70)/(1008-31.67^2)=1.5

a=70-1.5*31.67=22.5

AY 2016之最終損失金額=22.5+1.5*26=61.5(k)

(2)

a. If b is negative, then ultimate loss (Y) decreases when reported loss (x) increases.

b. If a is negative, then ultimate loss (Y) is negative when reported loss (x) is zero.

(3)

E(y)=(1-0.3)*70=49

d=31.67/70=0.452

VHM=(0.452^2)*(9^2)=16.58

EVPV=(0.08^2)*(9^2+49^2)=15.89

Z=VHM/(VHM+EVPV)=16.58/(16.58+15.89)=0.511

L(x)=0.511*26/0.452+(1-0.511)*49=53.33

4.（5 分）

下列資料係採用 GLM 配適損失發展三角形資料之結果:

Standardized Pearson Residuals

Accident Year	12 Months	24 Months	36 Months	48 Months	60 Months	72 Months
2011	-4.7	-2.78	3.37	1.68	2.65	0
2012	-1.91	-3.61	7.74	-1.34	-7.33	
2013	4.98	5.72	1.94	-6.91		
2014	0.12	-1.94	-1.22			
2015	-1.87	1.67				
2016	0					

Standard Deviations of Standardized Pearson Residuals

Accident Year	Standard Deviation	Accident Year Range	Standard Deviation
2011	3.571	2011 to 2012	4.463
2012	5.563	2013 to 2014	4.345
2013	5.797	2015 to 2016	2.503
2014	1.045	2011 to 2013	4.741
2015	2.503	2014 to 2016	1.537

Fitted Cumulative Losses

Accident Year	12 Months	24 Months
2013	4,000	8,521
2014	4,500	9,322
2015	6,000	11,554

An actuary reviewing the output notices heteroscedasticity in the residuals. The actuary decides to adjust for this by calculating variance parameters before running the sampling algorithm.

Given the following residual index values from one iteration of the sampling algorithm:

Accident Year	Sample Residual Index(row ,column)
2013	3,1
2014	5,2
2015	2,3

（1）請計算 sampled incremental losses for AY 2013-2015 between 12 and 24 months for the sample under consideration。（3 分）

（2）請說明為何 heteroscedastic residuals might cause issues when using a bootstrapping technique to estimate variance of unpaid claim estimates。（2 分）

【參考解答】

(1)

* Need to adjust residuals for heteroscedasticity.
* Group residuals by AY:
  2011 - 2013 have similar standard deviation
  2014 - 2015 have similar standard deviation

$m_{3,2}$=8521-4000=4521

$m_{4,2}$=9322-4500=4822

$m_{5,2}$=11554-6000=5554

AY	Unadjusted Sample Residual	AY Sampled From	Hetero-Adjustment	Adjusted r^*	$q^*(w,d)$
2013	4.98	2013	4.741/4.741	4.98	4856
2014	1.67	2015	1.537/1.537	1.67	4938
2015	7.74	2012	1.537/4.741	2.51	5741

$q^*(3,2) = 4.98 * 4521^{0.5} + 4521 = 4856$

$q^*(4,2) = 1.67 * 4822^{0.5} + 4822 = 4938$

$q^*(3,2) = 2.51 * 5554^{0.5} + 5554 = 5741$

(2)

The bootstrapping process assumes we can sample residuals from anywhere in the triangle. If the variance of residuals differs then our assumption of independent residuals is not valid. Adjusting for this keeps us from having overstated or understated estimated incremental losses during each iteration (depending on how the variance in that cell relates to other cells) and keeps the bootstrap variance of loss estimate from being artificially distorted.

5.（4 分）

依據下列 2016 年底之資料:

Policy Effective Year	Policy Effective Quarter	Ultimate Loss	Losses Reported at Prior Retro Adjustment
2012	1	80,000	80,000
2012	2	60,000	60,000
2012	3	150,000	150,000
2012	4	75,000	73,000
2013	1	65,000	60,000
2013	2	50,000	44,000
2013	3	60,000	50,000
2013	4	65,000	50,000
2014	1	45,000	30,000
2014	2	40,000	20,000
2014	3	80,000	50,000
2014	4	65,000	35,000
2015	1	55,000	-
2015	2	45,000	-
2015	3	55,000	-
2015	4	65,000	-
2016	1	35,000	-
2016	2	30,000	-
2016	3	25,000	-
2016	4	10,000	-

Retro Adjustment Period	Selected PDLD Ratio	Percent Loss Emerged	Policy Period	Premiums Booked from Prior Adjustment	Premiums Booked as of December 31,2016
First	2.0	70.0%	2012	$500,000	$505,000
Second	0.8	15.0%	2013	$345,000	$350,000
Third	0.6	10.0%	2014	$330,000	$340,000
Fourth	0.4	4.0%	2015~2016	0	$450,000
Subsequent	0.0	1.0%			

請計算 2016 年底之 premium asset。

【參考解答】

CPDLD1=1.596

CPDLD2=0.653

CPDLD3=0.507

CPDLD4=0.320

Expected Future Loss = Ult - Loss Reported as of Prior

Expected Future Prem = Expected Future Loss × CPDLD Prem Asset = Expected Future Prem + Prior Booked - Current Booked

AY	Expected Future Loss	CPDLD	Future Premium	Premium Asset
2012	2,000	0.320	640	-4,360
2013	36,000	0.507	18,252	13,252
2014	95,000	0.653	62,035	52,035
2015~2016	320,000	1.596	510,720	60,720
Total				121,647

6.（3 分）

A 精算師檢視 over-dispersed Poisson bootstrapping model 之 residual plots。

A 精算師選擇檢視 plot of the residuals vs. development periods。

（1）請舉出其他 2 種 residual plots 可供精算師檢視。（0.5 分）

（2）請說明 2 種 residual plots 呈現之現象，A 精算師應進一步調整模型。（0.5 分）

（3）A 精算師檢視 plot of the residuals vs. development periods 時發現，發展較不完整期間 residual 具有較大之絕對值之現象。A 精算師聲明這是合理之現象，他認為因為發展較不完整期間之 incremental values 通常較大，造成較大之變異數。

請針對 A 精算師之聲明進行評論。（2 分）

【參考解答】

（1）

Sample Answers (any two of which would earn full credit)

• Residuals vs. Accident Year
• Residuals vs. Calendar Year
• Residuals vs. Size of Loss (Prior Cumulative, Expected Incremental, etc.)
• Normality Plot
• Box and Whisker Plot

（2）

• If the actuary notices that variance is not constant across all residuals.
• If the actuary notices that residuals are trending, so for example, early AYs have positive residuals and later AYs have negative residuals.

（3）

His reasoning is not sound. Each residual is divided by the square root its expected variance based on the ODP model. Therefore if there is still variation in spread of residuals we have unexpected changes in variance and need to make an adjustment to our model.

7.（3 分）

依據下列資料，請測試並說明相鄰之損失發展因子是否存在相關性。

Cumulative Paid Losses

Accident Year	12 Months	24 Months	36 Months	48 Months	60 Months	72 Months
2011	9,000	11,000	12,000	12,500	12,800	13,000
2012	15,000	20,000	21,000	22,600	23,000	
2013	12,500	18,000	22,000	22,500		
2014	14,000	18,000	21,000			
2015	13,000	21,000				
2016	15,000					

【參考解答】

LDF

	12-24	24-36	36-48	48-60	60-72
2011	1.222	1.091	1.042	1.024	1.016
2012	1.333	1.050	1.076	1.018	
2013	1.440	1.222	1.023		
2014	1.286	1.167			
2015	1.615				

12-24 & 24-36

12-24 LDF rank	12-24 LDF rank	Difference	Squared
4	3	1	1
2	4	2	4
1	1	0	0
3	2	1	1

S=1+4+0+1=6

$$T_k = 1 - \frac{s}{(n*(n^2-1)/6)}$$

T_{24}=1-(6/(4*15)/6)=0.983

24-36 & 36-48

24-36 LDF rank	36-48 LDF rank	Difference	Squared
2	2	0	0
3	1	2	4
1	3	2	4

S=4+4=8

T_{36}=1-(8/(3*8)/6)=0.944

36-48 & 48-60

36-48 LDF rank	48-60 LDF rank	Difference	Squared
2	1	1	1
1	2	1	1

S=1+1=2

T_{36}=1-(2/(2*3)/6)=-1

T=(0.983*3+0.944*2+(-1)*1)/6=0.6395

VAR(T)=1/((6-2)*(6-3)/2)=0.167

we use a threshold of 50%, which is the percentile range [25%, 75%]. Thus, the confidence interval is

CI=(-0.67*0.167^0.5, 0.67*0.167^0.5)=(-0.274,0.274)

The test statistic T= 0.6395 isn't within the confidence interval. Therefore, we do reject the Null Hypothesis that the adjacent LDFs are correlated.

8.（3 分）

依據下列資料並採用 90%信賴區間，請測試並說明是否存在曆年度趨勢現象。

Cumulative Paid Losses

Accident Year	12 Months	24 Months	36 Months	48 Months	60 Months	72 Months
2011	13,000	19,000	23,000	25,500	26,500	27,000
2012	16,000	21,000	26,000	28,000	29,000	
2013	11,000	18,500	22,000	23,500		
2014	14,000	23,000	27,000			
2015	12,000	21,000				
2016	16,000					

【參考解答】

LDF

AY	12-24	24-36	36-48	48-60	60-72
2011	1.462	1.211	1.109	1.039	1.019
2012	1.313	1.238	1.077	1.036	
2013	1.682	1.189	1.068		
2014	1.643	1.174			
2015	1.750				

Rank columns, calculate z=min(S,L) for each diagonal

AY	12-24	24-36	36-48	48-60
2011	S	L	L	L
2012	S	L	*	S
2013	L	S	S	
2014	*	S		
2015	L			

Diagonal	n	m	cn	E[zn]	Var[zn]	z
1	1	0				
2	2	0	0.5	0.5	0.25	1
3	3	1	0.75	0.75	0.188	0
4	2	0	0.5	0.5	0.25	1
5	4	1	0.75	1.25	0.438	1

Sum of E[zn]=0.5+0.75+0.5+1.25=3

Sum of Var[zn]=0.25+0.188++0.25+0.438=1.126

Z=1+0+1+1=3

The Confidence Interval is 3± 1.645 * √1.126=(1.25,4.75)

Since Z=3, and it is within the confidence interval, it does not appear there are calendar year effects in the triangle.

9.（5 分）

下列最終損失發展因子資訊為AY 2016年損失限額200,000元之資料，假設48個月損失完全發展完成:

Age	12	24	36	48
LDF	2.20	1.25	1.10	1.00

AY 2016年每一發展期間損失假設服從指數分配，平均數如下:

Age	12	24	36	48
平均數	15,000	35,000	45,000	55,000

假設AY年度損失趨勢為10%，CY年度損失趨勢為5%，AY 2013~2016年損失資料如下:

AY	2013	2014	2015	2016
Paid to Date	14,000	12,000	9,000	5,000

請計算AY2013~2016年損失限額100,000元之最終損失金額。

【參考解答】

AY	AY TREND
2013	1.000
2014	1.100
2015	1.210
2016	1.331

CY TREND	12 Months	24 Months	36 Months	48 Months
2013	1.000	1.050	1.103	1.158
2014	1.050	1.103	1.158	1.216
2015	1.103	1.158	1.216	1.276
2016	1.158	1.216	1.276	1.340

TREND	12 Months	24 Months	36 Months	48 Months
2013	1.000	1.050	1.103	1.158
2014	1.155	1.213	1.273	1.337
2015	1.334	1.401	1.471	1.544
2016	1.541	1.618	1.699	1.784

Unlimit Mean	12 Months	24 Months	36 Months	48 Months
2013	9,735	22,715	29,206	35,696
2014	11,244	26,236	33,733	41,229
2015	12,987	30,303	38,961	47,619
2016	15,000	35,000	45,000	55,000

Eg. for AY 2015, age 12 Months: 12987=15000*1.334/1.541

LEV (X=100,000)	12 Months	24 Months	36 Months	48 Months
2013				33,528
2014			31,992	37,583
2015		29,185		41,788
2016	14,981			46,072

Eg. for AY 2016, age 12 Months: 15981=15000*[1-(e^(-100000/15000))]

LEV (B)	12 Months	24 Months	36 Months	48 Months
2013				
2014				
2015				
2016	15,000	34,885	44,472	53,551

Eg. for AY 2016, age 12 Months: 15000=15000*[1-(e^(-200000/15000))]

Finally, we can calculate the LDFs & Ultimate loss

AY	Paid to Date (1)	LDF (2)	Ult (3)=(1)*(2)
2013	14000	1.0000	14000
2014	12000	1.0731	12877
2015	9000	1.1659	10493
2016	5000	1.8952	9476

Eg. for AY 2016, LDF: 1.8952=(46072/53551)(14981/15000)

10.（4 分）

某保險公司採用下列方式管理核保循環:當市場soft時降低市占率，當市場hard時增加市占率。

(1) 請列出並詳細說明 1 種公司可用來降低盈餘波動的資產管理策略。(2 分)

(2) 當上述資產管理策略執行時，請詳細說明 1 種會增加的風險。(2 分)

【參考解答】

(1)

Sample Answer 1

Invest more in high yielding assets such as equity and high yield corporate bonds during soft market, and invest in more conservative assets such as treasury during hard market. Because during soft market, company is taking on less insurance risk by reducing market share, so it makes to take on more asset risk, and the extra investment income would help offset the reduction in UW income. During hard market it's the other way around.

Sample Answer 2

Shift assets more to equities when market is soft and move to bonds when market is hard. Equities typically have higher returns than bonds, so they should help make up for the decrease in UW profit in soft market. Conversely, higher UW profit in hard market will be offset by lower investment returns from bond-heavy asset portfolio. Should smooth out annual earnings.

Sample Answer 3

During soft market, invest more in taxable bonds with higher returns. During hard market, invest more in tax exempt bonds with lower return. Justification:

1. During soft market, company suffers UW loss. The higher investment income can help offset the underwriting loss, improving performance.
2. During hard market, company with decent UW profit can use tax exempt bonds to pay less tax on the investment income from tax exempt bonds.

(2)

Sample Answer 1

Asset risk would increase during soft markets – equities are riskier than bonds; there's a risk that market prices would decline after you invest more heavily in equities.

Sample Answer 2

Investment risk/asset risk. This is risk that company may see a drop in asset value if there's a market downturn, because now the company is investing more in higher risk asset.

Sample Answer 3

Taxable bonds with higher returns might have a longer duration, which would increase the interest rate risk.

11.（3 分）

某保險公司正在建構 ERM 計畫並具有下列特性：

a. ERM 計畫將定期監控檢討。
b. ERM 模型只包含保險風險及財務風險。
c. 該公司未將汽車保險包含在 ERM 模型，因為公司認為該業務屬於短尾險種、業務比重低且已停止承作該險種業務。
d. ERM 模型只考量不利之情境，因為公司認為有利之情境對公司而言沒有風險。
e. 該公司同時承保個人性險種及商業性險種，並且分別由個人商品部及商業商品部分別承保與定價，因此公司之 ERM 模型針對個人性險種及商業性險種採用 2 個完全獨立之模型進行評估。

請就上述 ERM 計畫之各項特性，評論屬於優勢或劣勢(strength or weakness)。

【參考解答】

a.

Strength – The process should be dynamic, and ready to respond to changing conditions

b.

Weakness – Operational and strategic risk are important to consider, even if they are difficult to quantify

c.

Strength – Program should focus on key risks that are material to the company. Short tailed, low exposure, and in runoff all point towards this risk not being material to the company as a whole.

d.

Weakness – The model should account for the capability to exploit risk when the outcome is favorable

e.

Weakness – There could be interdependency between the two lines in the tail. Separate models would underestimate the tail correlation for extreme events.

12.（3 分）

依據 Siewert, "A Model for Reserving Workers Compensation High Deductibles"，請簡述以損失率法(loss ratio approach)估計溢額賠款的主要優點及缺點。

【參考解答】

優點：

(1) Loss ratio estimates can be consistently tied to pricing programs, at least at the outset

(2) The procedure also benefits from its reliance on a more credible pool of company and/or industry experience

(3) It is useful when no data is available or the data is immature

缺點：

(1) A loss ratio approach ignores actual emerging experience, which in some circumstances may differ significantly from estimated ultimate losses

(2) Not particular useful after several years of development

(3) May not properly reflect account characteristics, as development may emerge differently due to the exposures written

13.（4 分）

已知下列資訊：

金額單位：千元

意外年度 Accident Year	滿期純保費 Earned Risk Pure Premium	調整保費 Adjusted Premium	已付賠款 Paid Losses	未付賠款 Case Reserves	已報賠款延遲 Aggregate Reported Loss Lag
2012	135,300	156,400	124,800	7,000	95%
2013	162,000	182,500	118,000	15,000	90%
2014	217,100	233,600	116,600	24,000	80%
2015	239,400	255,200	65,500	29,500	65%
2016	273,000	282,100	32,700	35,000	35%

請以 Stanard-Buhlmann 法計算預期損失率及未報賠款準備金。

【參考解答】

ELR = (124.8+7+118+15+116.6+24+65.5+29.5+32.7+35) /
(156.4·95%+182.5·90%+233.6·80%+255.2·65%+282.1·35%) = 74.33%

IBNR = 74.33%·(156.4·(1-95%)+182.5·(1-90%)+ 233.6·(1-80%)+ 255.2·(1-65%)
+282.1·(1-35%)) = 256.792（百萬元）

14.（4 分）

針對下列回溯計費保單資訊，

基本保費係數 Basic premium factor	0.15
預期損失率 Expected loss ratio	65%
賠款轉換係數 Loss conversion factor	1.1
稅賦乘數 Tax multiplier	1.05
首次調整之賠款發生預期比率 Expected percentage of loss emerged for the first adjustment	80%
第二次回溯調整之增加損失限額比率 Incremental loss capping ratio for the second retro adjustment	0.85
首次回溯調整之損失限額比率 Loss capping ratio at the first retro adjustment	0.60

依據 Teng and Perkin, " Estimating the Premium Asset on Retrospectively Rated Policies "，

(1) 請計算首次回溯調整之保費發展對損失發展比率($PDLD_1$)。(2 分)

(2)請計算第二次回溯調整之保費發展對損失發展比率($PDLD_2$)。(2 分)

【參考解答】

(1) $PDLD_1 = BP / (SP \cdot ELR \cdot \%Loss_1) \cdot TM + (CL_1 / L_1) \cdot LCF \cdot TM$

$= 0.15 / (65\% \cdot 80\%) \cdot 1.05 + 0.60 \cdot 1.1 \cdot 1.05$

$= 0.996$

(2) $PDLD_2 = (CL_2 \prec CL_1) / (L_2 \prec L_1) \cdot LCF \cdot TM$

$= 0.85 \cdot 1.1 \cdot 1.05$

$= 0.982$

15.（3 分）

依據 Teng and Perkin, " Estimating the Premium Asset on Retrospectively Rated Policies "，關於回溯計費保單，請回答下列問題：

(1)請簡述何謂為保單資產（premium asset）。(1.5 分)

(2)請說明保單資產於資產負債表如何認列。(1.5 分)

【參考解答】

(1) On retrospectively rated policies, premium that the insurer expects to collect based on the expected ultimate loss experience, less the premium that the insurer has already booked, is called the premium asset. Many insurers call this the Earned But Not Reported premium (EBNR).

(2) The admitted portion of the premium asset appears on the balance sheet as the “Asset for Accrued Retrospective Premiums.”

16. （4 分）

依據 Goldfarb, “P&C Insurance Company Valuation”，

(1)請問 Discounted Cash Flow 評價方法的主要缺點為何？(2 分)

(2)Discounted Cash Flow 與 Abnormal Earnings 二種評價方法皆須計算 Terminal value，請問二種方法關於 Terminal value 假設的差異為何？(2 分)

【參考解答】

(1) To estimate free cash flows, the analyst must first forecast financial statements (income statements and balance sheets) according to a specific set of accounting standards (U.S. GAAP, U.S. Statutory or International Accounting Standards). Then, a variety of adjustments are made to the forecasts of net income to estimate the free cash flow. The resulting values for free cash flow (to equity) may then bear little resemblance to the forecasts that management is familiar with, such as the values used within the firm’s internal planning process, the financial results of peer companies or the forecasts of external analysts. This might make it difficult to assess the reasonableness of the forecasted free cash flows or estimate their future growth rates.

(2) In the DCF valuation approaches, the terminal value calculation usually assumes that the free cash flows will continue in perpetuity and often the amounts are assumed to grow at a constant rate. Abnormal earnings are less likely to continue in perpetuity and are more likely to decline to zero as new competition is attracted to businesses with positive abnormal earnings.

17.（4 分）

依據 Marshall et al., “A Framework for Assessing Risk Margins”，不確定性來源分為系統性風險（systemic risk）及獨立性風險（independent risk）二大類。

（1）針對上述二大類風險，請分別說明其二種細分之風險來源為何。（2 分）

（2）若僅採 Bootstrapping 法分析準備金風險，請問分析範圍未包含哪些細分之風險來源？（2 分）

【參考解答】

(1) For systemic risk:

Risks internal to the insurance liability valuation process, collectively referred to in this paper as internal systemic risk. This source of uncertainty encapsulates the extent to which the adopted actuarial valuation approach is an imperfect representation of a complex real life process. Model structure and adequacy, model parameterisation and data accuracy are all aspects of internal systemic risk. This source of uncertainty is alternatively referred to as model specification risk.

Risks external to the actuarial modelling process, collectively referred to in this paper as external systemic risk. Even if the valuation model is an appropriate representation of reality, as it exists today, future systemic trends in claim cost outcomes that are external to the modelling process may result in actual experience differing from that expected based on the current environment and trends.

For independent risk:

The random component of parameter risk, representing the extent to which the randomness associated with the insurance process compromises the ability to select appropriate parameters in the valuation models.

The random component of process risk being the pure effect of the randomness associated with the insurance process. Even if the valuation model was perfectly calibrated to reflect expected future outcomes, the volatility associated with the insurance process is likely to result in differences from the perfect expected outcomes.

(2) Bootstrapping is inadequate alone to capture internal systemic risk or external systemic risk, to the extent that this latter differs from the past.

18. （3 分）

依據 Patrik, “Reinsurance”，請列出再保險人賠款準備金的 6 個組成項目（components）。

【參考解答】

(1) Case reserves reported by the ceding companies

(2) Reinsurer additional reserves on individual claims

(3) Actuarial estimate of future development on Components (1) and (2)

(4) Actuarial estimate of pure IBNR

(5) Discount for future investment income

(6) Risk load

19.（3 分）

已知某公司高自負額勞工補償保險資訊如下：

- 完整保障保費 – Full coverage premium $87,000,000
- 完整保障預期損失率 –Full coverage expected loss ratio 60%
- 溢額比率 – Excess ratio (per-occurrence charge) 10%
- 集合比率 – Aggregate ratio (per-aggregate charge) 2%

請依據 Siewert, "A Model for Reserving Workers Compensation High Deductibles"，估計該高自負額勞工補償保險之最終賠款。

【參考解答】

$P \cdot E \cdot \chi + P \cdot E \cdot (1-\chi) \cdot \varphi$
$= 87,000,000 \cdot 60\% \cdot (10\% + (1-10\%) \cdot 2\%)$
$= 6,159,600$

20.（5 分）

根據以下至 2016 年之資訊回答下列問題：

意外年度	已付賠款
2013	12,000
2014	11,250
2015	14,750
2016	9,500
總數	47,500

＊ 損失增量之發展乃根據 over-dispersed Poisson 分配，其中 scaling factor σ^2 = 25,000。

＊ 預期損失發展的 growth function 為 G(x) = x/(x+10)，x 表示預期損失發生後之平均月份，x 為平均意外發生日至評估日之月數。

＊ 整體預估未決賠款的係數之標準差為 850,000。

(1)(2 分)

請用 LDF 法，以 10 年為 truncation point 計算所有意外年度之未付賠款預的 coefficient of variation。

(2)(3 分)

請說明若預估未付賠款的方法由 LDF 法轉變成 Cape Code 法，coefficient of variance 會朝什麼方向變動，並說明朝此方向變動之原因。

【參考解答】

(1) Xtruncated = 12 x 10 - 6 = 11

G(114) = 114 / (114 + 10) = 0.919

意外年度	已付賠款	到期月份	G(X)	預期最終沒款	未付賠款
2013	12,000	42	0.808	13,654	1,654
2014	11,250	30	0.750	13,785	2,535
2015	14,750	18	0.643	21,086	6,336
2016	9,500	6	0.375	23,281	13,781
總數	47,500			71,806	24,306

2013 年之最終賠款=12000/(0.808/0.919)=13,654

Process variance = σ^2 * reserve = 25000*24306 = 607,650,000

Total standard deviation = (850,000^2+607,650,000)^0.5 = 850,357

Coefficient of variance = 850357/24306=34.99

(2) Coefficient of variance 將會下降，因 Cape Cod 會採用更多資訊計算，並對於損失尚未發展完之年度使用較穩定之損失率，而非採用變動性較高之 LDF。

21.（5 分）

請根據以下損失發展因子回答下列問題：

損失發展因子				
意外年度	12-24個月	24-36個月	36-48個月	48-60個月
2011	1.600	1.375	1.091	1.125
2012	5.000	1.100	2.000	
2013	2.833	1.588		
2014	2.091			

假設 $T = r\,[(n-2)/(1-r^2)]^{½}$

下列表格為 t-statistic for 0.9 在不同的 degrees of freedom：

Degrees of Freedom	1	2	3
t-statistic	6.314	2.920	2.354

請用 Venter's correlation test at 10% level 測試 12-24 個月與 24-36 個月的發展因子間的相關性是否顯著。

【參考解答】

$$r = \frac{E[XY] - E[X] \times E[Y]}{\sigma_X \times \sigma_Y}$$

	12-to-24 Months	24-to-36 Months		(X - E[X])²	(Y - E[Y])²
AY	X	Y	XY	2.3850	0.0004
2011	0.6000	0.3750	0.2250	3.4435	0.0647
2012	4.0000	0.1000	0.4000	0.0969	0.0546
2013	1.8330	0.5880	1.0778	5.9254	0.1197
Mean	2.1443	0.3543	0.5676	n 3	3
				Variance 1.9751	0.0399
				Standard Dev. 1.4054	0.1998

$$r = \frac{0.5676 - 2.1443 \times .3543}{(1.4054 \times 0.1998)} \quad r = -0.6846$$

n=3, Degrees of freedom = n-2 = 1 所以 t-statistic=6.314

$$T = r \times [(n-2)/(1-r^2)]^{.5} = -0.6846 \times [(3-2)/(1-(-0.6846)^2)]^{.5}$$

= -0.9393 < 6.314 因此兩個期間發展因子的相關性並不顯著。

22.(3 分)
若一專門承保財產險之保險公司須購買再保險以分保其承保之巨災風險,保險公司有以下兩個再保險選擇，並且兩個選擇提供同樣的價格與承保一樣的風險。
選擇 A：再保公司的業主權益為 10 億元，主要業務為財產險之再保險，並僱有最高等的信評。
選擇 B：再保公司的業主權益為 10 億元，主要業務為意外險之再保險，並僱有最高等的信評。

(1) 請解釋兩個選擇不同的風險程度，並說明兩個選擇對於採用資本模型評估所需股東權益時有何差別。(1.5 分)
(2) 請說明為何將承保範圍與風險 100%分出給單一再保公司，對保險公司的資金運用不是最佳的選擇。(1.5 分)

【參考解答】
(1) 選擇 B 對保險公司是一個較好的選擇，因為當巨災發生時將同時影響專營財產險的保險公司與再保公司，因此將影響再保公司的損失攤回能力，但若購買來自主要業務為意外險之再保險公司，較具有風險分散的效果。因此選擇 A 的風險較高，將會要求較高之風險邊際，所需的股東權益也相對較高。
(2) 假設再保險公司間的財務狀況並不完全相關，若購買單一再保險公司提供之再保，將有較大的信用風險，因此不是最佳選擇。

23.（5 分）

根據以下某再保險公司截至 12/31/2015 為止之資料回答下列問題：

Calendar/ Accident Year	Earned Risk Pure Premium	Adjusted Premium	Aggregate Reported Loss	Reported loss lag	Chain Ladder IBNR
2011	12,500	13,000	11,500	95%	500
2012	13,300	13,800	10,000	85%	1,500
2013	14,000	14,200	8,500	72%	3,100
2014	15,500	15,800	7,900	55%	4,500
2015	16,000	16,000	6,000	40%	7,000
總數	71,300	72,800	43,900		16,600

(1)請採用 Standard Buhlmann 方法計算所有意外年度之 IBNR。(1 分)

(2)請分別描述若再保人採用 Standard-Buhlmann 方法計算 IBNR 之優點與缺點。(2 分)

(3)請採用 credibility-weighted estimate 的方法結合 chain ladder 法和 Standard-Buhlmann 法計算 IBNR，其中 chain ladder 法的 credibility factor 為 0.8。(2 分)

【參考解答】

(1) SB ELR = 43900/(13000*95%+13800*85%+14200*72%+15800*55%+16000*40%) = 88.9%

SB IBNR = 88.9% * (13000*5%+13800*15%+14200*28%+15800*45%+16000*60%) = 20,808

(2) 優點：採用實際損失資料計算 ELR，而非如 BF 法採用主觀判斷。

缺點：須調整每年的保費以反映過去的費率變更。

(3) Credibility-weighted IBNR =

(0.8*95%*500+(1-0.8*95%)*578+0.8*85%*1500+(1-0.8*85%)*1840+0.8*72%*3100+

(1-0.8*72%)*3535+0.8*55%*4500+(1-0.8*55%)*6321+0.8*40%*7000+(1-0.8*40%)*8534)

= 18,975

24.(3 分)

精算師在建立隨機 chain ladder model 時，考慮以下之分配：

* Over-dispersed Poisson
* Over-dispersed Negative Binomial
* Normal

實際 2016 年意外年度之損失發展至 12 個月= 60,000

以 chain ladder model 預估 2016 年意外年度之損失發展至 24 個月= 80,000

(1)（1.5 分） 根據以下之損失分配模型，計算 2016 年意外年度之損失發展至 24 個月之變異數(variance)。

Over-dispersed Poisson model； ϕ=1.5

Over-dispersed Negative Binomial model； ϕ=1.25

Normally distributed model； ϕ=1.75

(2)（1.5 分） 精算師想採用與 chain ladder 法有顯見連結之模型，請說明以上三個模型哪個適用。

【參考解答】

(1) Over-dispersed Poisson model：80000*(1/1-1/1.5)*1.5=40,000

Over-dispersed Negative Binomial model：1.25*(1.5-1)*1.5*60000=56,250

Normally distributed model：1.75*60000=105,000

(2) * Negative Binomial：其公式與 chain ladder 較接近

* Normal：其持續性的機率分配支持(-∞,+∞)

25.(4 分)

(1) 請根據以下保險公司之財務資訊，依 Free Cash Flow to Equity(FCFE)計算公司於 1/1/2018 之價值。(2 分)

	2018	2019	2020
期初股東權益	1,000		
淨損益	80	100	140
為維持於年底之信評AA之最低資本需求	1015	1035	1040
於年底依法規之最低資本需求	813	840	875
於年底為達成管理階層鎖定之成長目標之最低資本需求	1017	1015	1035
賠款準備金之變動	100	-75	25
淨借貸金額	12	0	15

- 預估 equity market risk premium = 6%
- 無風險利率(risk-free rate) = 2%
- 保險公司之 beta（β）= 1.25
- 採用 CAPM 決定風險調整後之折現率(risk-adjusted discount rate)

(2) 請說明採用 FCFE 方法預估公司價值之缺點。(2 分)

【參考解答】

(1)

Required return = 2% + 1.25 * 6% = 9.5%

FCFE = Net income + Non-cash charges (excluding changes in reserves) - Net working capital investment - Increase in required capital + Net borrowing

在每年年底選擇最高之資本要求

	2018	2019	2020
淨損益	80	100	140
期初股東權益	1,000	1017	1035
期末股東權益	1017	1035	1040
淨借貸金額	12	0	15
資本變動	17	18	5
FCFE	75	82	150
ROE	8.0%	9.8%	13.5%
Reinvestment = Δ股東權益/淨損益	0.2125	0.1800	0.0357
Growth = ROE * Reinvestment	0.017	0.018	0.005

平均 growth rate = (0.017+0.018+0.005)/3 = 0.013

公司價值= $\frac{75}{1.095}+\frac{82}{1.095^2}+\frac{150}{1.095^3}+\frac{\frac{150*1.013}{0.095-0.013}}{1.095^3}$ = 1662.5

(2) 計算中有一大部分是來自於成長率與折現率的預估。

預估過程需調整財務報表，對於公司管理階層較難聊解。

26.(4 分)

請根據以下追溯費率之保險合約之資訊，計算 Retrospective premium asset。

保單生效年	保單生效季	最終損失	在前一期調整後的已報損失
2012	1	92,500	92,500
2012	2	57,000	57,000
2012	3	125,000	125,000
2012	4	80,000	78,000
2013	1	64,000	57,000
2013	2	37,500	30,000
2013	3	60,000	55,000
2013	4	65,000	6,000
2014	1	55,000	50,000
2014	2	40,000	33,000
2014	3	70,000	52,500
2014	4	60,000	45,000
2015	1	50,000	0
2015	2	40,000	0
2015	3	65,000	0
2015	4	45,000	0
2016	1	25,000	0
2016	2	20,000	0
2016	3	30,000	0
2016	4	5,000	0

回溯調整期	選擇的PDLD Ratio	Percent Loss Emerged
1	1.75	78.5%
2	0.70	10.0%
3	0.55	7.0%
4	0.45	4.0%
4以後	0.00	0.5%

Policy Period	Premiums Booked from Prior Adjustment	Premiums Booked as of December 31, 2016
2012	450,000	452,000
2013	335,000	337,000
2014	330,000	335,000
2015-2016	-	425,000

【參考解答】

$$\text{CPDLD} = \frac{\sum \text{PDLD} \times \%\ \text{Loss Emerged}}{\sum \%\ \text{Loss Emerged}}$$

回溯調整	CPDLD
1	1.500
2	0.588
3	0.491
4	0.400

Est Future Prem = (Ult loss – loss reported at prior) × CPDLD of next adj

Year	Next Retro is	Estimated Future Premium
2012	4	800
2013	3	12,037
2014	2	26,183
2015	1	300,050
2016	1	120,020

Premium Asset = Premium from prior adjustment + Estimated future premium – premium booked

= 450000+335000+330000+800+12037+26183+300050+120020-452000-337000-335000-425000

= 25090

（試題結束）