2017 G6 高等費率釐定

計算及申論題共計 20 題：

Q1（4 分）

假設一年內可能發生之四起事件資訊如下，且各事件間彼此獨立，每一事件只會發生一次或不發生，試回答下列問題：

事件	機率	損失金額
1	0.10	50,000
2	0.03	100,000
3	0.02	150,000
4	0.07	125,000

(1)請計算年平均損失(Average Annual Loss)。(1 分)

(2)請說明發生超越機率(Occurrence Exceedance Probability ,OEP)之定義。(1 分)

(3)請計算每一損失檔級之發生超越機率(OEP)。(2 分)

【參考解答】

(1)年平均損失
(AAL)=0.1*50,000+0.03*100,000+0.02*150,000+0.07*125,000=19,750

(2)The OEP is the probability that at least one loss exceeds the specified loss amount.

(3)各損失檔級 OEP 如下表所示 ：

事件	機率	損失金額	OEP
3	0.02	150,000	0
4	0.07	125,000	0.02
2	0.03	100,000	0.0886
1	0.10	50,000	0.1159
None		0	0.2043

Q2（6 分）

一精算師於非比例性再保合約採用下列 Exposure Curve 及相關資訊進行評估：

$$G(x) = \frac{(1-b^x)}{(1-b)}$$

Maximum Possible Loss	\$5,000,000
Insured Value	\$5,000,000
Gross Premium	\$6,000
Expected Loss Ratio	60%
Retention of non-proportional reinsurance treaty	\$150,000
Expected Ceded Risk Premium	\$2,750

(1) 給定全損(Total loss)機率為 0.03，請計算上述公式中之參數 b 為何?(2 分)

(2) 承上題，請計算此非比例再保合約之限額(limit)為何?(4 分)

【參考解答】

(1) $p = \frac{G\prime(1)}{G\prime(0)} = (\frac{-\ln(b)b^1}{1-b})/(\frac{-\ln(b)b^0}{1-b})$=b=0.03

(2) Expected risk premium=6,000*60%=3,600

2,705=3,600*[G((150k + Lim)/5M) - G(150k/5M)]

G(150/5000)=G(0.03)=(1-0.03^0.03)/(1-0.03)=0.103

G((150k + Lim)/5M)=2,705/3,600+0.103=0.854=(1-0.03^x)/(1-0.03)

1-0.854*0.97=0.03^x=0.171

X=0.503=(150k+Lim)/5M

Limit=2,365,000(2,365,769)

Q3（5 分）

下表為再保人對原保險人契約之累積損失分佈：

損失率區間	區間平均損失率	區間損失機率
0-50%	44%	6%
50%-75%	67%	64%
75%-100%	86%	22%
>100%	109%	8%

原保險人對於損失率介於 75%至 100%之間的損失將承受 80%

(1)請計算再保險人經過損失調整前的預期損失率。(2.5 分)

(2)請計算再保險人經過損失調整後的預期損失率。(2.5 分)

【參考解答】

(1) Expected Gross Loss Ratio=(6%)(44%)+(64%)(67%)+(22%)(86%)+(8%)(109%)= 73.16%

(2) Expected LR net of corridor=(6%)(44%)+(64%)(67%)+(22%)(**77.2%**)+(8%)(**89%**)= 69.624%

- 77.2%=75%+(20%)(86%-75%)
- 89.0%=75%+(20%)(100%-75%)+(109%-100%)

Q4（5 分）

某被保險人群體具有不同期望損失頻率之被保險人，被保險人過去 t 年無肇事紀錄資料如下：

期望損失頻率	t=0	t=1	t=2	t=3
0.05	45000	45500	44000	43000
0.10	46000	43500	44000	35000
0.20	23000	19500	16300	13500
合計	114000	108500	104300	91500

請評估 whether the variation of an individual insured’s chance for an accident changes over time。

【參考解答】

	(1)	(2)	(3)	(4)	(5)
n	# Claim free n or more years	Expected Claims	Frequency	Relative Frequency	Z
3	91500	8350	0.0913	0.9499	0.0501
2	195800	18210	0.0930	0.9681	0.0319
1	304300	28735	0.0944	0.9830	0.0170
TOTAL	418300	40185	0.0961	1.0000	0.0000

Expected claims:

- t=3: 43,000 x 0.05 + 35,000 x 0.10 + 13,500 x 0.20 = 8,350
- t=2: 44,000 x 0.05 + 44,000 x 0.10 + 16,300 x 0.20 = 9,860
- t=1: 45,500 x 0.05 + 43,500 x 0.10 + 19,500 x 0.20 = 10,525
- t=0: 45,000 x 0.05 + 46,000 x 0.10 + 23,000 x 0.20 = 11,450

(3) = (2)/(1)

(4) = (3)/(3)Total

(5) = 1 - (4)

If the variation of an insured’s chance for an accident is not changing over time, then the 3-year credibility/1-year credibility will be approximately equal to 3 and the 2-year credibility/1-year credibility will be approximately equal to 2.

3+ year Z / 1+ year Z = 0.0501/ 0.0170 = 2.94

2+ year Z / 1+ year Z = 0.0319 / 0.0170 = 1.87

The ratios are approximately 3 and 2; the chance for accident is stable.

Q5（6 分）

某精算師使用 GLM 分析車險歷史資料：

(1)請針對下列因子納入 GLM 分析提出反對的意見：(3 分)

a. Limit of liability.
b. Number of coverage changes during the current policy period.
c. ZIP code of the garaging location of the automobile.

(2)精算師 modeling pure premium with a log-link function and a Tweedie error distribution（$1<p<2$）。請說明 2 項反對將自負額納入此模型之理由。(1.5 分)

(3)若不將自負額納入 GLM 模型中直接評估，請提出決定自負額係數的方法並說明決定之自負額係數如何融入於 GLM 模型中。(1.5 分)

【參考解答】

(1)

Sample Responses for [a]

• Including limit of liability in the GLM can lead to counterintuitive results such as lower relativity for higher limit due to correlation with other variables not included in the model.

• Including limit may give unexpected results like lower rate for more coverage due to adverse or favorable selection.

Sample Responses for [b]

• The information will not be available for new business since we are building a GLM for the prospective period.

• Number of coverage changes is likely to change from what it is in the current policy period and thereafter year by year.

Sample Responses for [c]

• Too many ZIP codes to include it in the GLM; using a spatial smoothing technique would be more appropriate and include the determined value for ZIP code as an offset term in the GLM.

• Sparse data creates credibility concerns and it will add too many degrees of freedom to the model.

• There are too many ZIP codes to be used in a GLM. Furthermore, aggregating them into groups will cause a great loss of information.

• Too many ZIP codes create too many parameters which will potentially lead to overfitting

(2)

•Deductibles should lower frequency (small losses below deductible not reported) but increase severity (since claims that do get reported are higher average cost). This violates the assumption for Tweedie that variables move frequency and severity in the same direction.

• Deductible factors may produce higher relativities at higher deductibles due to factors other than pure losses elimination:

1. Insureds at high loss potential and high premiums may elect high deductibles to reduce premium 2. Underwriters may force high deductibles on high risks

• Deductible factors are likely correlated with other factors outside of the model and may give non intuitive results like paying more for less coverage; for example because underwriters force high risk insureds to purchase higher deductibles.

(3)

•The deductible relativities can be calculated using a mix of experience and exposure rating and then included in the GLM model as an offset.

• Determine deductibles relativities by means loss elimination calculation with historical data [i.e., portion of loss not paid because of deductible E(x;d)/E(x)]. Include the relativities as an offset term in the GLM.

• Deductible relativities should be determined based purely of loss elimination, outside of the GLM model. Then they should be included as offset factors in the log-link function as +ln(relativity).

Q6（4.5 分）

某精算師正在開發網路資料保護責任保險並且採用產業別作為費率因子，不同的商業類型(business types)例如:餐廳、汽車製造工廠、牧場將會分到不同的產業分類(different industry groups)中，該精算師享用幾個產業因子(industry factors)將不同商業類型 (business types)分類至產業分類(industry groups)。

(1)Describe a benefit that a principal components method would have over a generalized linear model for determining the industry factors.（1.5分）

(2)Briefly describe the major steps in using a cluster analysis to group the industry factors.（1.5 分）

(3)Describe two test statistics that could be used to determine the optimal number of groups from the cluster analysis. Identify which statistic would be preferred when variables are correlated.(1.5 分)

【參考解答】

(1)

A principal components analysis will identify which variables are most predictive of the variance between businesses, allowing any other highly correlated variables to be removed, resulting in a simpler model.

(2)

i. Select the number of clusters.

ii. Provide some initial assignment of businesses to clusters.

iii. Compute the centroid (the average industry factor) of each cluster.

iv. Calculate the distance from each business to each cluster (i.e., the difference in the industry factor).

v. For each business, assign the business to the cluster with the closest centroid (the closest industry rating factor).

vi. Repeat steps (iii) and later if any businesses moved to a different cluster.

(3)

i. The Calinski and Harabasz statistic: measures the between varaince of the clusters divided by the within variance.

ii. The Cubic Clustering Criterion (CCC) statistic: Compares the variance explained by a given set of clusters to that expected when clusters are formed at random based on the multi-dimensional uniform distribution.

The CCC test statistic is less reliable when variables are correlated, so the Calinski and Harabasz statistic would be preferred.

Q7（4 分）

某精算人員想評估下列三種experience rating plans何者最佳。每一個Plan都測試四個不同之風險，測試結果如下所示:

Plan 1		
Risk Number	Predicted Modification Factor	Error
1	1.25	30%
2	1.25	30%
3	0.75	20%
4	0.75	20%

Plan 2		
Risk Number	Predicted Modification Factor	Error
5	1.25	20%
6	1.25	-20%
7	0.75	15%
8	0.75	-15%

Plan 3		
Risk Number	Predicted Modification Factor	Error
9	1.25	8%
10	1.1	5%
11	0.9	-5%
12	0.75	-8%

請分別依據Meyers/Dorweiler criterion and及least squared error criterion決定並說明何者為最佳之experience rating plan。

【參考解答】

The least squared error criterion would choose the plan with the smallest sum of the squared errors. Based on the given errors for each plan, Plan 3 will best satisfy this criterion.

The Meyers/Dorweiler criterion would choose the plan with no correlation between the modification factors and the errors. Plan 1 has larger errors that correspond with larger modification factors, so there is a correlation. Plan 3 has positive errors for larger modification factors and negative errors for smaller modification factors, so again there is a correlation. Plan 2 exhibits no correlation between errors and modification factors, so Plan 2 will best satisfy this criterion.

Q8（5.5 分）

某精算人員正在研究一種新方法用來估計某個州的損失頻率時，用來分配給觀察值的可信度方式，精算人員將經驗資料分為Test data及Holdout sample並計算該方法的Quintile test結果如下表。

Quintile	Holdout Relativity	Prediction Based on Countrywide Average	Prediction Based on Raw Test Data	Prediction Based on Credibility Procedure
1	0.50	1.00	0.30	0.85
2	0.80	1.00	0.40	0.90
3	0.90	1.00	0.80	0.95
4	1.20	1.00	1.40	1.05
5	1.50	1.00	2.00	1.15
Mean	1.00	1.00	1.00	1.00
SSE		0.655	0.550	0.425

(1)請說明holdout sample的目的。(1.5分)

(2)假設精算人員蒐集2007~2016年的損失頻率資料，請提供一種適當的選擇holdout sample 的方式。(1.5分)

(3)請評估此新方法高估或低估該州係數(state relativities) 之可信度。(1.5分)

(4) 請評估此新方法及採用全國平均何者較適當用來計算州係數該(state relativities)。(1 分)

【參考解答】

(1)A holdout sample is a split of the original dataset that was not used to build the model and is instead used for testing the predictive power of the model.

(2)An appropriate holdout sample would be to use the odd years of data (i.e., 2007, 2009, 2011, 2013, and 2015). Then the even years would be used to build the model. This will make sure both datasets are equally impacted by seasonality or trends over time.

(3)For states with low frequency relativities, the new procedure estimates state relativities that too high relative to the holdout data (e.g., 0.85 > 0.5), so the procedure gives too little credibility to the raw data estimates (e.g., 0.3) and too much credibility to the countrywide estimate (1.00). Similarly, for states with high frequency relativities, the new procedure estimates state relativities that too low relative to the holdout data (e.g., 1.15 < 1.50), so the procedure again gives too little credibility to the raw data estimates (e.g., 2.00) and too much credibility to the countrywide estimate (1.00). Therefore, the new procedure consistently understates the credibility of the raw state relativity data.

(4)Since the SSE when predicting holdout data using the credibility procedure is lower than the SSE for the countrywide average (and the raw data), the new credibility procedure will be more accurate and should be used to predict state relativities.

Q9（8 分）

某險別共有 200 個保單，個別保單整體賠款(aggregate losses)的分布情形如下：

合計賠款	保單件數
10,000	12
20,000	30
30,000	40
50,000	36
60,000	22
70,000	26
80,000	12
90,000	2
100,000	20

請依據上述統計結果建立包含 Insurance charge $\phi(r)$ 的 Table M，其中 Entry ratios 範圍為 0 到 2.0，並以 0.2 遞增計算。

【參考解答】

平均賠款＝10*12+20*30+…+100*20=50(K)

Entry Ratio	Number of Risks at Ratio	Risk÷Total Risks	Risks at Higher Ratios	Losses in Next Layer	Charge
0.00	0	0.000	1.000	0.200	1.000
0.20	12	0.060	0.940	0.188	0.800
0.40	30	0.150	0.790	0.158	0.612
0.60	40	0.200	0.590	0.118	0.454
0.80	0	0.000	0.590	0.118	0.336
1.00	36	0.180	0.410	0.082	0.218
1.20	22	0.110	0.300	0.060	0.136
1.40	26	0.130	0.170	0.034	0.076
1.60	12	0.060	0.110	0.022	0.042
1.80	2	0.010	0.100	0.020	0.020
2.00	20	0.100	0.000	0.000	0.000

Q10 (3 分)

依據 Venter, "Experience Rating－Equity and Predictive Accuracy"，請問採用經驗費率法的二個主要目的為何？

【參考解答】

1 安全誘因 (Safety incentive)：

依據發生之危險事故對被保險人收費，對安全性提供了財務上的誘因。

2 預測準確性 (Predictive accuracy)：

經驗費率調整在某種程度上可預測未來損失經驗，可使保費更貼近被保險人個別的損失可能情況。

Q11(6 分)

某車體損失險目前無自負額之規定，現欲新增扣減式自負額 (straight deductible)之選擇。請依據下列資訊，計算自負額為$30,000 時的費率占原費率的比率。

賠款金額級距	賠款件數	總賠款金額(千元)
0 – 10,000	1,628	10,500
10,000 – 30,000	3,056	71,000
30,000 – 50,000	1,015	43,500
50,000 – 100,000	878	73,500
> 100,000	268	45,550

項目	%	是否隨總保費變動
賠款(含可分配理賠費用)	65	否
可分配理賠費用	3	否
佣金	15	是
稅	3	是
一般費用	15	否
利潤	2	是

【參考解答】

$k = (L_r + (N-n)r) / L$

= (10,500+71,000+(1,015+878+268)*30)/(10,500+…+45,550)

= 0.60

D = k(E-a) / (1-A-T-p)

= 0.6*(.65-.03) / (1-.15-.03-.02)

= 0.465

$Rate_{30K}$ = 1 - .465 = 0.535

Q12 (3 分)
依據 Miccolis, "On the Theory of Increased Limits and Excess of Loss Pricing"，且根據下列資料，

每次事故限額	E[g(x; k)]	$E[g(x; k)^2]$
$ 300,000	50,000	$4(10^9)$
500,000	77,000	$5(10^{10})$
1,000,000	129,000	$6(10^{11})$

- 基本限額為$300,000
- 損失頻率為 Poisson 分配
- $\lambda = 10^{-8}$

(1) 請計算各保額加計風險考量(risk loading)之高保額係數。 (1.5 分)
(2) 若損失頻率為 Binomial 分配，請說明各保額加計之風險考量會增加或減少。(1.5 分)

【參考解答】

(1) $I(500,000) = (77000+10^{-8}\times5(10^{10})) / (50000+10^{-8}\times4(10^{9})) = 1.55$
$I(1,000,000) = (129000+10^{-8}\times6(10^{11})) / (50000+10^{-8}\times4(10^{9})) = 2.70$

(2) 若為 Binomial dist.,
$\mathrm{Var}(y) = E(n) \cdot \mathrm{Var}(g(x)) + \mathrm{Var}(n) \cdot E(g(x))^2 < E(n) \cdot E(g(x)^2)$
故加計之風險考量會減少。

Q13（3 分）

Gillam and Snader 列舉了三個可信度應符合的條件，請分別以數學式及文字敘述說明此三個條件。

【參考解答】

(1) $0 \leqq Z \leqq 1$

Credibility should not be less than zero and not greater than unity

(2) $dZ/dE \geqq 0$

Credibility should not decreases as the size of risk increases

(3) $d(Z/E)/dE<0$

As the size of risk increases, the percentage charge for a loss of a given size should decrease

Q14 (6 分)

Given the following pure premium information about a block of auto policies:

以下為某群車險保單之純保費相關資訊：

Attachment Point 起賠點	Excess Losses as a Percent of Total 超額損失佔總損失的比率	Excess Losses per Exposure Unit 每暴險單位之超額損失
$0	100%	4,000
$25,000	75%	3,000
$100,000	50%	2,000
$500,000	35%	1,400
$1,000,000	10%	400

a. （1 分）

Calculate the pure premium for coverage from $500,000 to $1,000,000. Show all work.

請計算投保範圍為$500,000 到$1,000,000 的損失的純保費。

b. （3 分）

Given that $25,000 is the basic limit for a policy, calculate the four increased limits factors for the policy limits of $25,000, $100,000, $500,000, and $1,000,000. Show all work.

假設$25,000 是基本限額，請計算$25,000, $100,000, $500,000, 與$1,000,000 等保額之高保額係數。

c. （2 分）

Do the increased limits factors calculated in part (b) above pass the consistency test as described by Miccolis? Explain why or why not.

請問(b)之高保額係數有符合 Miccolis 的一致性測試嗎?請簡述原因。

【參考解答】

a. 1,400 － 400 ＝ 1,000

b. 25,000 高保額係數= 1.00（基本限額）

限額 25,000 的純保費 ＝ 4,000 － 3,000 ＝ 1,000

$100,000 高保額係數 ＝（4,000 － 2,000）／ 1,000 ＝ 2.00

$500,000 高保額係數 ＝（4,000 － 1,400）／ 1,000 ＝ 2.60

$1,000,000 高保額係數 ＝（4,000 － 400）／ 1,00 ＝ 3.60

c.

保額	高保額係數	測試
$25,000	1.00	
$100,000	2.00	(2.00 – 1.00) / (100 – 25) = 0.0133
$500,000	2.60	0.0015
$1,000,000	3.60	0.002

未通過一致性測試，因為高保額係數沒有隨保額增加而邊際遞減。
$500,000 到$1,000,000 邊際增加 0.002 > $100,000 到$5000,000 邊際增加 0.0015

Q15 (7 分)

Using the following set of data for a group of large risks, determine whether the current or proposed experience rating plan is better. Assume the risks are all of the same premium size.

利用以下大額損失的分組資料，請說明現在的經驗損失計畫與擬採用的經驗損失計畫哪個比較好(假設所有風險都有相同的保費量)：

Risk	Current Plan Mod	Manual Loss Ratio	Current Plan Standard Loss Ratio	Proposed Plan Mod	Proposed Plan Standard Loss Ratio
A	0.80	0.65	0.86	0.85	0.90
B	0.85	0.70	0.86	0.97	0.80
C	0.87	0.85	1.18	1.03	1.17
D	0.92	0.90	0.80	0.72	0.95
E	0.94	0.94	0.95	0.77	1.10
F	0.99	0.95	0.82	0.75	0.97
G	1.00	0.92	0.95	1.08	0.92
H	1.07	0.99	0.96	1.08	0.97
I	1.09	1.04	0.95	0.99	1.05
J	1.12	1.01	1.08	1.30	1.13

【參考解答】

Manual LR 的平均數	0.895
Manual LR 的變異數	0.014905
現行計畫 Standard LR 的平均數	0.941
現行計畫 Standard LR 的變異數	0.012469
現行計畫效率測試值 Efficiency Test Statistic = (0.012469 / 0.014905)	0.837
擬採用計畫 Standard LR 的平均數	0.996
擬採用計畫 Standard LR 的變異數	0.011884
擬採用計畫效率測試值 Efficiency Test Statistic = (0.011884 / 0.014905)	0.797

效率測試值較低者為較佳的計畫，因 0.797< 0.837，擬採用計畫較佳

Q16（4 分）

The following information is available for a LDD policy:

以下是關於某 LDD 保單的資訊：

標準保費 Standard premium	$1,200,000
預期最終損失率 Expected ultimate loss ratio	0.80
州與風險分組相對因子 State hazard group relativity	1.1
自負額 Deductible	$200,000
超額損失因子 Excess loss factor	0.24
總和損失 Aggregate limit on deductible	$772,800

以下是該保單 Table M 的資訊：

Expected Loss Group 期望損失組別	Range Rounded Values 金額範圍
30	$600,001 - $750,000
29	$750,001 - $925,000
28	$925,001 - $1,100,000
27	$1,100,001 - $1,300,000
26	$1,300,001 - $1,600,000
25	$1,600,001 - $1,950,000
24	$1,950,001 - $2,200,000

	Expected Loss Group 期望損失組別						
Entry ratio	30	29	28	27	26	25	24
0.75	0.4069	0.3989	0.3911	0.3833	0.3755	0.3677	0.3599
0.81	0.3777	0.3690	0.3605	0.3521	0.3436	0.3352	0.3267
1.07	0.2764	0.2661	0.2557	0.2453	0.2349	0.2245	0.2141
1.15	0.2522	0.2417	0.2310	0.2203	0.2096	0.1989	0.1882
1.23	0.2347	0.2241	0.2134	0.2027	0.1920	0.1813	0.1706
1.53	0.1690	0.1583	0.1476	0.1369	0.1261	0.1154	0.1047

2017 G6 高等費率釐定

請採用 Insurance Charge Reflecting Loss Limitation (ICRLL)過程計算期望損失成本。

【參考解答】

k = 0.24 / 0.80 = 0.30

調整後的預期損失=1,200,000×0.80×1.1×((1+0.8×0.30)/(1-0.30)=1,870,629

以調整後的預期損失查表 ELG 為第 25 組

預期超額損失= ($1,200,000)(0.24) = $288,000

預期限額下損失= ($1,200,000×0.80) - $288,000 = $672,000

r_G= $772,800 / $672,000 = 1.15

以 1.15 查表(第 25 組)得到 insurance charge = 0.1989

預期損失 = $288,000 + (0.1989)($672,000) = $421,661

Q17 (5 分)

Let X represent the size of loss of a given claim. Assume the following average severities for varying claim size ranges:

令 X 等於損失金額，下表為 X 在不同區間的平均數：

Size of Loss Range 損失金額範圍	Average Size of Loss 損失金額平均數
X <= $1,000	$500
$1,000 < X <= $2,000	$1,500
X > $2,000	$4,000

The following information is also available:

另有以下資訊：

總賠案數 Total number of claims in study	500
損失金額平均數 Overall average claim size	$1,300
安全係數 Safety factor	0.80
自負額$1,000 的 loss elimination ratio Tempered loss elimination ratio for a $1,000 straight deductible	0.40

請計算自負額$2,000 的 loss elimination ratio。 (Calculate the tempered loss elimination ratio for a straight deductible of $2,000.)

【參考解答】

令損失金額小於$1,000 的賠案數為 A

$0.4=0.8\times\frac{500A+1000(500-A)}{500\times1300}$

求出 A=350

令損失金額介於 1,000 到$2,000 的賠案數為 B

$1300=\frac{500\times350+1500B+4000(500-350-B)}{500}$

求出 B=50

損失金額超過 \$1,000 的賠案數為 500 - 350 - 50 = 100

Tempered PLER at \$1,000= $0.80\times\frac{(500)(350)+(1500)(50)+(2,000)(100)}{500\times1300}$=0.55

Q18(6 分)

考量以下資訊，為超額勞工補償保險保單定價：

- Excess loss pure premium factors are based on empirical data for losses and ALAE up to $250,000 and a fitted curve for losses greater than $250,000
  超額純保費因子計算方式：$250,000 以下由損失加可分配理賠費用之經驗資料計算，$250,000 以上由損失配適曲線求得
- 起賠點為$1,000,000 (attachment point)
- 無總和限額(No aggregate limit)

相同大小之風險的調整後歷史資料如下：

損失金額	機率
$20,000	70%
100,000	14%
250,000	8%
500,000	5%
750,000	2%
1,000,000	1%

標準保費	$500, 000
不可分配理賠費用佔損失與可分配理賠費用的百分比	4%
一般費用	6%
招攬費用	15%
稅	3%
利潤率	2%

- Empirical data has been truncated and shifted at $250,000 and normalized to a unity mean.
  經驗資料經過截取(truncated)與平移(shifted)至$250, 000，且標準化至相同的平均數
- A mixed Exponential-Pareto curve has been fit to the resulting mean residual lives as described by the following parameters:
  採用 Exponential-Pareto 曲線來配適，參數如下：

分配	Pareto	Exponential
Cumulative Function	$1+(1+\frac{x}{b})^{-s}$	$1-e^{-x/c}$
Mean	$\frac{b}{s-1}$	C
Variance	$\frac{b^2}{(s-1)^2(s-2)}$	C^2

2017 G6 高等費率釐定

Excess Ratio	$(1+\frac{x}{b})^{1-s}$	$e^{-x/c}$
Mean Residual Life	$\frac{b+x}{s-1}$	C
Shape	4.0	n/a
Scale	12.0	0.8
Weight	0.050	0.950

請求出該保單之保費。

【參考解答】

E[A] = (20k)(0.70) + (100k)(0.14) + (250k)(0.08) + (500k)(0.05) + (750k)(0.02) + (1M)(0.01) = 98k

250k以上之預期損失(Exp Loss above 250k) = (500k - 250k)(0.05) + (750k - 250k)(0.02) + (1M - 250k)(0.01) = 30k

當損失超過250k時的預期損失(Exp Loss above 250k conditional on a loss being above 250k) = 30k / (0.05 + 0.02 + 0.01) = 375k

曲線配適的Entry ratio(Entry ratio for fitted curve)= (1M - 250k) / 375k = 2

R(1M) = R_{data}(250k) × R_{fit}(2)

R_{data}(250k) = 30k / 98k = 0.306

Pareto分配的平均數 = 12/(4-1) = 4

Exponential分配的平均數= 0.8

$R_{\text{Pareto}}(2) = (1 + 2/12)^{(1-4)} = 0.6297$

$R_{\text{Exponential}}(2) = e^{(2/0.8)} = 0.0821$

$R_{fit}(2) = \frac{(0.05)(4)(0.6297)+(0.95)(0.8)(0.0821)}{(0.05)(4)+(0.95)(0.8)} = 0.1962$

R(1M) = 0.306 × 0.1962 = 0.0601

保費 $= \frac{(98,000)(0.0601)(1+4\%)+(6\%)(500,000)}{1-15\%-3\%-2\%} = \$45,157$

Q19 (3 分)

一回溯型的費率保單同時有損失限額與最高保費限制

a. (1 分)

請以圖形說明損失限額與最高保費限制會造成 charges 重疊(overlap)

b. (1 分)

Explain how the overlap is handle differently when using Table M versus Table L.

請說明 Table M 與 Table L 在處理重疊(overlap)的相異處。

c. (1 分)

大型勞工補償保險的訂價，其固定損失限額為$100,000，請問你建議使用 Table M 還是 Table L 來計算 charges，請提出兩個理由。

【參考解答】

a.

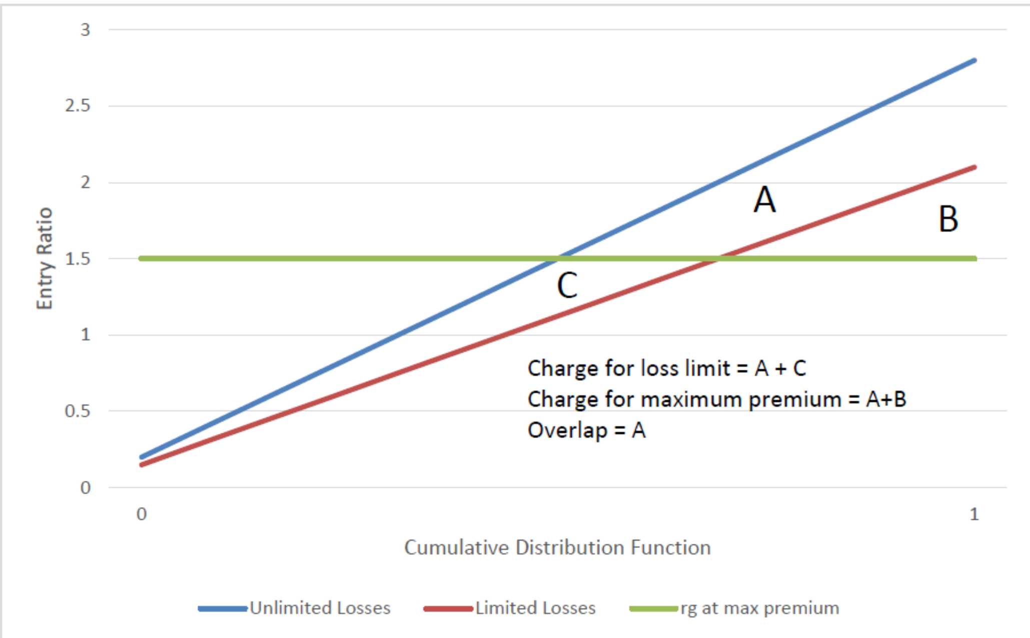
Draw a Lee Diagram labeling unlimited loss curve, limited loss curve, entry ratio at max premium, and indicating where the charges overlap.

下圖分別畫出未受限額的損失曲線、受限額的損失曲線、最高保費的entry ratio與標示出the charges overlap

損失限額的charges = A+C

最高保費限額的charges = A+B

Overlap=A



When there is an occurrence limit as well as an aggregate limit, the occurrence limit makes it less likely that the aggregate limit will be hit. Thus, the charge for the aggregate limit should be reduced, otherwise there will be overlap.

當每事故限額與總損失限額都存在時，每事故限額會降低損失金額超過總損失限額的機率，所以總損失限額的charge要降低，不然會造成overlap。

b.

關於Table M(任一)

- Adjust expected loss to a larger size to approximate a limited table M

  調整大型客戶的預期損失，來近似有限額的Table M
- It uses the AEL procedure to make aggregate loss distribution less skewed approximating the overlap correction

  採用AEL程序使總額損失的機率分配降低偏態以近似overlap的修正。
- Uses the ICRLL procedure to shift the curve to approximate a limited loss curve

  採用ICRLL程序來移動曲線到近似於有限額的損失曲線。

關於Table L(任一)

- Charges are calculated with both an occurrence and aggregate limit

  採用同時有每事故限額與總損失限額的資料來計算Charges
- Builds tables for separate limits on capped losses

  建立不同每事故限額與總限額的表格。

c.

關於Table M(任二)

- Based on countrywide data so more credible

  採用全國資料，可信度較高
- More easily updated for inflation by adjusting ELG table

  調整ELG表格，較容易調整通貨膨脹之影響
- More flexible for changing loss limits from year to year as you do not need a separate table for each limit

  對於每年改變的損失限額較有彈性，不需要每種限額都要有一張表
- Table L is built using California taxes so not appropriate for use in other states

  Table L採用加州的資料製作，不適合用在別州

關於Table L

- Table L provides more accurate estimation for the insurance charge

Table L對insurance charge的估計較為準確

- Since there is a fixed loss limit there is not a need for a large number of tables to accommodate changing limits
  因為有固定的損失限額，所以不需要對不同的限額製作很多張表。

Q20（6 分）

某公司想要重新檢討費率計畫，讓費率因子包含年齡組別，以下為基礎模型(base model)與新模型(new model)的各種數據

數據	基礎模型	新模型
Loglikelihood	-1,500	-1,475
Deviance	1,000	950
Parameters	20	30
Data Points	2,000,000	2,000,000

a. （4分）
請計算兩種模型的Akaike Information Criterion(AIC)與Bayesian Information Criterion (BIC)。

b. （1分）
請說明AIC與BIC哪個是比較可靠的分辨哪個模型較適合的指標。

c. （1分）
請建議是否應該採用新模型並說明理由。

【參考解答】

a.

Sample 1

AIC = -2LL + 2p

BIC = -2LL + plog(n)

Base Model:

AIC = -2(-1500) + 2(20) =3020

BIC = -2(-1500) + 20log(2M) = 3126

New Model:

AIC = -2(-1475) + 2(30) = 3010

BIC = -2(-1475) + 30log(2M) = 3139

Sample 2

AIC Base = -2(-1500) + 2(20) =3020

AIC New = -2(-1475) + 2(30) = 3010

BIC Base = -2(-1500) + 20ln(2,000,000) =3290

BIC New = -2(1475) + 30ln(2,000,000) = 3385

Sample 3

AIC = D+2p

AIC Base = 1000 + 2(20) = 1040

AIC New = 950 + 2(30) =1010

BIC = D + pln(n)

BIC Base = 1000 + 20ln(2,000,000) =1290

BIC New = 950+ 30ln(2,000,000) = 1385

Sample 4

AIC = Deviance + 2p

AIC Base = 1000 + 2(20) = 1040

AIC New = 950 + 2(30) =1010

BIC = Deviance + plog(n)

BIC Base = 1000 + 20x log(2,000,000) = 1126

BIC New =950+ 30 x log(2,000,000) = 1139

b.（任一）

- 資料點很多，使用BIC的話會被ln(data points)影響，故建議採用AIC。
- AIC較適合，因為保險模型通常來自大量資料，使用BIC的話嚴重的被多餘的參數數量懲罰並且通常會得到剃除額外參數的建議。

c.（任一）

- 建議採用，因新模型的AIC較低(新模型的BIC較高，但是採用AIC當指標較合適)
- BIC New > BIC Base.新模型的BIC較高，故不建議採用新模型
- 新模型的AIC只有稍低一點，但是新模型的BIC較高，故不建議採用新模型

(試題結束)