G5 高等準備金，價值評估與企業風險管理

1. (3 分)

某保險公司正在建構 ERM 計畫並具有下列特性：

a. ERM 計畫將定期監控檢討。

b. ERM 模型只包含保險風險及財務風險。

c. 該公司未將汽車保險包含在 ERM 模型，因為公司認為該業務屬於短尾險種、業務比重低且已停止承作該險種業務。

d. ERM 模型只考量不利之情境，因為公司認為有利之情境對公司而言沒有風險。

e. 該公司同時承保個人性險種及商業性險種，並且分別由個人商品部及商業商品部分別承保與定價，因此公司之 ERM 模型針對個人性險種及商業性險種採用 2 個完全獨立之模型進行評估。

請就上述 ERM 計畫之各項特性，評論屬於優勢或劣勢(strength or weakness)。

【參考解答】

a. Strength - The process should be dynamic, and ready to respond to changing conditions

b. Weakness - Operational and strategic risk are important to consider, even if they are difficult to quantify

c. Strength - Program should focus on key risks that are material to the company. Short tailed, low exposure, and in runoff all point towards this risk not being material to the company as a whole.

d. Weakness - The model should account for the capability to exploit risk when the outcome is favorable

e. Weakness - There could be interdependency between the two lines in the tail. Separate models would underestimate the tail correlation for extreme events.

【題目出處】

Brehm :Enterprise Risk Analysis for Property & Liability Insurance Companies

2. (6 分)

(1) 當在 ERM 中採用 default avoidance 做為資本要求設定之參考點時，請詳細說明 2 項缺點。(2 分)

(2) 請另外舉出並說明 2 種可做為資本要求設定之參考點。(2 分)

(3) 請選擇 1 種第(2)小題中之參考點，develop a minimum capital requirement that relates a maximum capital loss tolerance to a TVaR measurement. (2 分)

【參考解答】

(1)

Sample Answer 1

Default is a very unlikely outcome in the far tail of the distribution of outcomes. The ERM model is probably not very accurate at this point in the distribution so using default avoidance as a reference point may not yield accurate results from the ERM model. Default avoidance mainly protects policyholder. However, other stakeholders (e.g., Shareholders) may care about large partial decreases in capital. To protect all stakeholders, need to choose more likely reference point than default.

Sample Answer 2

This requires selecting a capital level deep in the tail of the loss distribution, which is exactly where the loss distribution is least reliable. Default avoidance mainly protects policyholders. Shareholders can be hurt at losses lower than default level. Thus a lower level than default level may be more meaningful for the firm.

Sample Answer 3

Shareholders are impacted by a loss in value before the company is close to default. Capital requirements should consider protecting shareholders Default usually happens far out into the tail of a loss distribution where the results may not be as credible. Capital requirements should be based on a credible estimate.

(2)

Sample Answer 1

Sufficient capital to continue servicing renewals. Sufficient capital to withstand and thrive after a catastrophe.

Sample Answer 2
Rating agency requirement- what level of capital is required to maintain rating Point at which capital could support renewal book

Sample Answer 3
Setting capital at a level that maximizes franchise value Setting capital at a level to service renewal book

(3)
Sample Answer 1
Suppose renewals are 80% of the book, so we want to minimize the chance that we will lose more than 02% of our capital in a given year. We want to set our capital = 5×TV@R90% this means one out of 10 years were are expected to lose an amount of capital equal to TV@R90% which is TV@R/ (5×TV@R90%) = 20%, so we can still service renewals.

Sample Answer 2
No more than 20% of capital to 1 an 100 event (needed to maintain capital to service ongoing business). Minimum Capital Requirement (MCR) = TV@R90% X 5. TV@R90% is the expected value of a 1 in 100 event. If that occurred, we would lose TV@R90% / (5 ×TV@R90%) = 20% of capital.

Sample Answer 3
To hold enough capital to not only survive a major CAT but thrive in its aftermath; Set minimum capital equal to 6 time 95th percentile TV@R. This ensures that an average 1 in 20 year event will deplete only 1/6th of the company’s capital. So, even after this event the company will not just survive, but should have enough remaining capital to thrive.

【題目出處】
Brehm :Enterprise Risk Analysis for Property & Liability Insurance Companies

3. (4 分)

依據下列 2017 年底之資料:

Accident Year	Cumulative Reported Loss @ 24 Months	Ultimate Loss
2014	$30,000	$70,000
2015	$35,000	$75,000
2016	$30,000	$65,000
2017	$26,000	

(1) 請使用 least squares method 計算 AY 2017 之最終損失金額. (1 分)

(2) 請針對下列情境，舉出並說明一個採用 least squares method 面臨的問題。(1 分)

a. The slope parameter is negative.

b. The intercept parameter is negative.

(3) 請考量下列情境之改變。(2 分)

a. No change in the reporting pattern.

b. Standard deviation of reported loss as of 24 months will be 8% of estimates ultimate loss.

c. Expected ultimate loss for AY 2016 will decrease 30%.

d. Standard deviation of AY 2016 ultimate loss is expected to be $9,000.

請使用 Bayesian credibility method 計算 AY 2016 之最終損失金額。

【參考解答】

(1)

$\bar{X}$=(30+35+30)/3=31.67

$\bar{Y}$=(70+75+65)/3=70

$\overline{XY}$=(30*70+35*75+30*65)/3=2225

$\overline{X^2}$=(30^2+35^2+30^2)/3=1008

b=(2225-31.67*70)/(1008-31.67^2)=1.5

a=70-1.5*31.67=22.5

AY 2016之最終損失金額=22.5+1.5*26=61.5(k)

(2)

a. If b is negative, then ultimate loss (Y) decreases when reported loss (x) increases.

b. If a is negative, then ultimate loss (Y) is negative when reported loss (x) is zero.

(3)

E(y)=(1-0.3)*70=49

d=31.67/70=0.452

VHM=(0.452^2)*(9^2)=16.58

EVPV=(0.08^2)*(9^2+49^2)=15.89

Z=VHM/(VHM+EVPV)=16.58/(16.58+15.89)=0.511

L(x)=0.511*26/0.452+(1-0.511)*49=53.33

4. (5 分)

下列資料係採用 GLM 配適損失發展三角形資料之結果:

Standardized Pearson Residuals

Accident Year	12 Months	24 Months	36 Months	48 Months	60 Months	72 Months
2011	-4.7	-2.78	3.37	1.68	2.65	0
2012	-1.91	-3.61	7.74	-1.34	-7.33	
2013	4.98	5.72	1.94	-6.91		
2014	0.12	-1.94	-1.22			
2015	-1.87	1.67				
2016	0					

Standard Deviations of Standardized Pearson Residuals

Accident Year	Standard Deviation	Accident Year Range	Standard Deviation
2011	3.571	2011 to 2012	4.463
2012	5.563	2013 to 2014	4.345
2013	5.797	2015 to 2016	2.503
2014	1.045	2011 to 2013	4.741
2015	2.503	2014 to 2016	1.537

Fitted Cumulative Losses

Accident Year	12 Months	24 Months
2013	4,000	8,521
2014	4,500	9,322
2015	6,000	11,554

An actuary reviewing the output notices heteroscedasticity in the residuals. The actuary decides to adjust for this by calculating variance parameters before running the sampling algorithm.
Given the following residual index values from one iteration of the sampling algorithm:

Accident Year	Sample Residual Index(row ,column)
2013	3,1
2014	5,2
2015	2,3

(1) 請計算 sampled incremental losses for AY 2013-2015 between 12 and 24 months for the sample under consideration. (3 分)

(2) 請說明為何 heteroscedastic residuals might cause issues when using a bootstrapping technique to estimate variance of unpaid claim estimates. (2 分)

【参考解答】

(1)

* Need to adjust residuals for heteroscedasticity.
* Group residuals by AY:
  2011 - 2013 have similar standard deviation
  2014 - 2015 have similar standard deviation

$m_{3,2}$=8521-4000=4521

$m_{4,2}$=9322-4500=4822

$m_{5,2}$=11554-6000=5554

AY	Unadjusted Sample Residual	AY Sampled From	Hetero-Adjustment	Adjusted r^*	$q^*(w,d)$
2013	4.98	2013	4.741/4.741	4.98	4856
2014	1.67	2015	1.537/1.537	1.67	4938
2015	7.74	2012	1.537/4.741	2.51	5741

$q^*(3,2) = 4.98 * 4521^{0.5} + 4521 = 4856$

$q^*(4,2) = 1.67 * 4822^{0.5} + 4822 = 4938$

$q^*(3,2) = 2.51 * 5554^{0.5} + 5554 = 5741$

(2)

The bootstrapping process assumes we can sample residuals from anywhere in the triangle. If the variance of residuals differs then our assumption of independent residuals is not valid. Adjusting for this keeps us from having overstated or understated estimated incremental losses during each iteration (depending on how the variance in that cell relates to other cells) and keeps the bootstrap variance of loss estimate from being artificially distorted.

5.（4 分）

依據下列 2016 年底之資料:

Policy Effective Year	Policy Effective Quarter	Ultimate Loss	Losses Reported at Prior Retro Adjustment
2012	1	80,000	80,000
2012	2	60,000	60,000
2012	3	150,000	150,000
2012	4	75,000	73,000
2013	1	65,000	60,000
2013	2	50,000	44,000
2013	3	60,000	50,000
2013	4	65,000	50,000
2014	1	45,000	30,000
2014	2	40,000	20,000
2014	3	80,000	50,000
2014	4	65,000	35,000
2015	1	55,000	-
2015	2	45,000	-
2015	3	55,000	-
2015	4	65,000	-
2016	1	35,000	-
2016	2	30,000	-
2016	3	25,000	-
2016	4	10,000	-

Retro Adjustment Period	Selected PDLD Ratio	Percent Loss Emerged	Policy Period	Premiums Booked from Prior Adjustment	Premiums Booked as of December 31,2016
First	2.0	70.0%	2012	$500,000	$505,000
Second	0.8	15.0%	2013	$345,000	$350,000
Third	0.6	10.0%	2014	$330,000	$340,000

Fourth	0.4	4.0%	2015~2016	0	$450,000
Subsequent	0.0	1.0%			

請計算 2016 年底之 premium asset。

【參考解答】

CPDLD1=1.596
CPDLD2=0.653
CPDLD3=0.507
CPDLD4=0.320

Expected Future Loss = Ult - Loss Reported as of Prior
Expected Future Prem = Expected Future Loss × CPDLD Prem Asset = Expected Future Prem + Prior Booked - Current Booked

AY	Expected Future Loss	CPDLD	Future Premium	Premium Asset
2012	2,000	0.320	640	-4,360
2013	36,000	0.507	18,252	13,252
2014	95,000	0.653	62,035	52,035
2015~2016	320,000	1.596	510,720	60,720
Total				121,647

6. (3 分)

A 精算師檢視 over-dispersed Poisson bootstrapping model 之 residual plots。

A 精算師選擇檢視 plot of the residuals vs. development periods。

(1) 請舉出其他 2 種 residual plots 可供精算師檢視。(0.5 分)

(2) 請說明 2 種 residual plots 呈現之現象，A 精算師應進一步調整模型。(0.5 分)

(3) A 精算師檢視 plot of the residuals vs. development periods 時發現，發展較不完整期間 residual 具有較大之絕對值之現象。A 精算師聲明這是合理之現象，他認為因為發展較不完整期間之 incremental values 通常較大，造成較大之變異數。
請針對 A 精算師之聲明進行評論。(2 分)

【參考解答】

(1)

Sample Answers (any two of which would earn full credit)

- Residuals vs. Accident Year
- Residuals vs. Calendar Year
- Residuals vs. Size of Loss (Prior Cumulative, Expected Incremental, etc.)
- Normality Plot
- Box and Whisker Plot

(2)

- If the actuary notices that variance is not constant across all residuals.
- If the actuary notices that residuals are trending, so for example, early AYs have positive residuals and later AYs have negative residuals.

(3)

His reasoning is not sound. Each residual is divided by the square root its expected variance based on the ODP model. Therefore if there is still variation in spread of residuals we have unexpected changes in variance and need to make an adjustment to our model.

7.（3 分）

依據下列資料，請測試並說明相鄰之損失發展因子是否存在相關性。

Cumulative Paid Losses

Accident Year	12 Months	24 Months	36 Months	48 Months	60 Months	72 Months
2011	9,000	11,000	12,000	12,500	12,800	13,000
2012	15,000	20,000	21,000	22,600	23,000	
2013	12,500	18,000	22,000	22,500		
2014	14,000	18,000	21,000			
2015	13,000	21,000				
2016	15,000					

【參考解答】

LDF

	12-24	24-36	36-48	48-60	60-72
2011	1.222	1.091	1.042	1.024	1.016
2012	1.333	1.050	1.076	1.018	
2013	1.440	1.222	1.023		
2014	1.286	1.167			
2015	1.615				

12-24 & 24-36

12-24 LDF rank	12-24 LDF rank	Difference	Squared
4	3	1	1
2	4	2	4
1	1	0	0
3	2	1	1

S=1+4+0+1=6

$$T_k = 1 - \frac{s}{(n*(n^2-1)/6)}$$

T_{24}=1-(6/(4*15)/6)=0.983

24-36 & 36-48

24-36 LDF rank	36-48 LDF rank	Difference	Squared
2	2	0	0
3	1	2	4
1	3	2	4

S=4+4=8

T_{36}=1-(8/(3*8)/6)=0.944

36-48 & 48-60

36-48 LDF rank	48-60 LDF rank	Difference	Squared
2	1	1	1
1	2	1	1

S=1+1=2

T_{36}=1-(2/(2*3)/6)=-1

T=(0.983*3+0.944*2+(-1)*1)/6=0.6395

VAR(T)=1/((6-2)*(6-3)/2)=0.167

we use a threshold of 50%, which is the percentile range [25%, 75%]. Thus, the confidence interval is

CI=(-0.67*0.167^0.5, 0.67*0.167^0.5)=(-0.274,0.274)

The test statistic T= 0.6395 isn't within the confidence interval. Therefore, we do reject the Null Hypothesis that the adjacent LDFs are correlated.

【題目出處】

Mack(1994) : Variability of Chain Ladder Reserve Estimates

8.（3 分）

依據下列資料並採用 90%信賴區間，請測試並說明是否存在曆年度趨勢現象。

Cumulative Paid Losses

Accident Year	12 Months	24 Months	36 Months	48 Months	60 Months	72 Months
2011	13,000	19,000	23,000	25,500	26,500	27,000
2012	16,000	21,000	26,000	28,000	29,000	
2013	11,000	18,500	22,000	23,500		
2014	14,000	23,000	27,000			
2015	12,000	21,000				
2016	16,000					

【參考解答】

LDF

AY	12-24	24-36	36-48	48-60	60-72
2011	1.462	1.211	1.109	1.039	1.019
2012	1.313	1.238	1.077	1.036	
2013	1.682	1.189	1.068		
2014	1.643	1.174			
2015	1.750				

Rank columns, calculate z=min(S,L) for each diagonal

AY	12-24	24-36	36-48	48-60
2011	S	L	L	L
2012	S	L	*	S
2013	L	S	S	
2014	*	S		
2015	L			

Diagonal	n	m	cn	E[zn]	Var[zn]	z
1	1	0				
2	2	0	0.5	0.5	0.25	1
3	3	1	0.75	0.75	0.188	0

4	2	0	0.5	0.5	0.25	1
5	4	1	0.75	1.25	0.438	1

Sum of E[zn]=0.5+0.75+0.5+1.25=3

Sum of Var[zn]=0.25+0.188++0.25+0.438=1.126

Z=1+0+1+1=3

The Confidence Interval is 3± 1.645 * √1.126=(1.25,4.75)

Since Z=3, and it is within the confidence interval, it does not appear there are calendar year effects in the triangle.

【題目出處】

Mack(1994) : Variability of Chain Ladder Reserve Estimates

9.（5 分）

下列最終損失發展因子資訊為AY 2016年損失限額200,000元之資料，假設48個月損失完全發展完成:

Age	12	24	36	48
LDF	2.20	1.25	1.10	1.00

AY 2016年每一發展期間損失假設服從指數分配，平均數如下:

Age	12	24	36	48
平均數	15,000	35,000	45,000	55,000

假設AY年度損失趨勢為10%，CY年度損失趨勢為5%，AY2013~2016年損失資料如下:

AY	2013	2014	2015	2016
Paid to Date	14,000	12,000	9,000	5,000

請計算AY2013~2016年損失限額100,000元之最終損失金額。

【參考解答】

AY	AY TREND
2013	1.000
2014	1.100
2015	1.210
2016	1.331

CY TREND	12 Months	24 Months	36 Months	48 Months
2013	1.000	1.050	1.103	1.158
2014	1.050	1.103	1.158	1.216
2015	1.103	1.158	1.216	1.276
2016	1.158	1.216	1.276	1.340

TREND	12 Months	24 Months	36 Months	48 Months
2013	1.000	1.050	1.103	1.158
2014	1.155	1.213	1.273	1.337
2015	1.334	1.401	1.471	1.544
2016	1.541	1.618	1.699	1.784

Unlimit Mean	12 Months	24 Months	36 Months	48 Months
2013	9,735	22,715	29,206	35,696
2014	11,244	26,236	33,733	41,229
2015	12,987	30,303	38,961	47,619
2016	15,000	35,000	45,000	55,000

Eg. for AY 2015, age 12 Months: 12987=15000*1.334/1.541

LEV (X=100,000)	12 Months	24 Months	36 Months	48 Months
2013				33,528
2014			31,992	37,583
2015		29,185		41,788
2016	14,981			46,072

Eg. for AY 2016, age 12 Months: 15981=15000*[1-(e^(-100000/15000))]

LEV (B)	12 Months	24 Months	36 Months	48 Months
2013				
2014				
2015				
2016	15,000	34,885	44,472	53,551

Eg. for AY 2016, age 12 Months: 15000=15000*[1-(e^(-200000/15000))]

Finally, we can calculate the LDFs & Ultimate loss

AY	Paid to Date (1)	LDF (2)	Ult (3)=(1)*(2)
2013	14000	1.0000	14000
2014	12000	1.0731	12877
2015	9000	1.1659	10493
2016	5000	1.8952	9476

Eg. for AY 2016, LDF: 1.8952=(46072/53551)(14981/15000)

【題目出處】

Sahasrabuddhe : Claims Development by Layer

10. (4 分)
某保險公司採用下列方式管理核保循環:當市場soft時降低市占率，當市場hard時增加市占率。

(1) 請列出並詳細說明 1 種公司可用來降低盈餘波動的資產管理策略。(2 分)
(2) 當上述資產管理策略執行時，請詳細說明 1 種會增加的風險。(2 分)

【參考解答】

(1)

Sample Answer 1
Invest more in high yielding assets such as equity and high yield corporate bonds during soft market, and invest in more conservative assets such as treasury during hard market. Because during soft market, company is taking on less insurance risk by reducing market share, so it makes to take on more asset risk, and the extra investment income would help offset the reduction in UW income. During hard market it's the other way around.

Sample Answer 2
Shift assets more to equities when market is soft and move to bonds when market is hard. Equities typically have higher returns than bonds, so they should help make up for the decrease in UW profit in soft market. Conversely, higher UW profit in hard market will be offset by lower investment returns from bond-heavy asset portfolio. Should smooth out annual earnings.

Sample Answer 3
During soft market, invest more in taxable bonds with higher returns. During hard market, invest more in tax exempt bonds with lower return.
Justification:
1. During soft market, company suffers UW loss. The higher investment income can help offset the underwriting loss, improving performance.
2. During hard market, company with decent UW profit can use tax exempt bonds to pay less tax on the investment income from tax exempt bonds.

(2)

Sample Answer 1
Asset risk would increase during soft markets - equities are riskier than bonds; there's a risk that market prices would decline after you invest more

heavily in equities.

Sample Answer 2
Investment risk/asset risk. This is risk that company may see a drop in asset value if there's a market downturn, because now the company is investing more in higher risk asset.

Sample Answer 3
Taxable bonds with higher returns might have a longer duration, which would increase the interest rate risk.

【題目出處】
Brehm :Enterprise Risk Analysis for Property & Liability Insurance Companies

11.（3 分）

已知某保險公司下列資訊：

金額單位：百萬元

年度	期初股東權益	淨利
2017	3,500	280
2018	3,700	330
2019	3,800	380
2020	3,900	390

- 該公司 beta 為 1.6
- 期望市場報酬率為 5%
- 無風險利率為 1.5%
- 評估該公司 Abnormal Earnings 自 2020 年後每年減少 1/2。

請依據 Goldfarb, “P&C Insurance Company Valuation” 所述之 Abnormal Earnings Valuation Method 評估該公司權益價值(total equity value)。

解答：

年度	期初股東權益	淨利	期望報酬	Abnormal Earning	PVAE
2017	3,500	280	248.5	31.5	29.4
2018	3,700	330	262.7	67.3	58.7
2019	3,800	380	269.8	110.2	89.7
2020	3,900	390	276.9	113.1	86.0

$K = r_f + \beta (r_m - r_f) = 1.5\% + 1.6 \cdot (5\% - 1.5\%) = 7.1\%$

該公司 2016 年底權益價值

$= 3,500 + 29.4 + 58.7 + 89.7 + 86.0 + 113.1 \cdot (1/2)/(7.1\% + 1/2)/1.071^4$

$= 3,839.0$（百萬）

12（3 分）

已知下列資訊：

金額單位：千元

意外年度	滿期總保費	滿期純保費	調整純保費	已報賠款	預期損失率	已報賠款延遲(Report Lag)
2013	547,000	330,000	379,000	294,000	50%	95%
2014	651,000	409,000	458,000	257,000	52%	85%
2015	627,000	388,000	419,000	253,000	55%	75%
2016	553,000	337,000	353,000	192,000	60%	60%
2017	680,000	433,000	433,000	147,000	58%	35%

- 可信度因子(credibility factor)為 0.6

請依據 Patrik, “Reinsurance”，計算可信度 IBNR 估計值(Credibility IBNR Estimate)。

解答：

SBELR = (294+…+147) / (379·95%+…+433·35%) = 78.9%

CL IBNR	SB IBNR	Z	"Credibility" IBNR
15,474	14,952	0.57	15,249
45,353	54,204	0.51	49,690
84,333	82,648	0.45	83,406
128,000	111,407	0.36	117,380
220,500	204,982	0.24	208,706

15,474 = 294,000·(1/95%-1)

14,952 = 379,000·78.9%·(1-95%)

0.57 = 0.6·95%

15,249 = 15,474·0.57 + 14,952·(1-0.57)

13. （4 分）

已知下列資訊：

金額單位：千元

意外年度 Accident Year	滿期純保費 Earned Risk Pure Premium	調整保費 Adjusted Premium	已付賠款 Paid Losses	未付賠款 Case Reserves	已報賠款延遲 Aggregate Reported Loss Lag
2012	135,300	156,400	124,800	7,000	95%
2013	162,000	182,500	118,000	15,000	90%
2014	217,100	233,600	116,600	24,000	80%
2015	239,400	255,200	65,500	29,500	65%
2016	273,000	282,100	32,700	35,000	35%

請以 Stanard-Buhlmann 法計算預期損失率及未報賠款準備金。

解答：

ELR = (124.8+7+118+15+116.6+24+65.5+29.5+32.7+35) /
(156.4·95%+182.5·90%+233.6·80%+255.2·65%+282.1·35%) = 74.33%

IBNR = 74.33%·(156.4·(1-95%)+182.5·(1-90%)+ 233.6·(1-80%)+ 255.2·(1-65%)
+282.1·(1-35%)) = 256.792（百萬元）

14. **(4 分)**

依據下列資料:

Accident Year	Cumulative Paid Losses($000) 12 Months	24 Months	36 Months
2013	2,500	4,100	5,000
2014	2,600	4,000	
2015	2,800		

假設AY loss emergence pattern (growth function)服從Weibull 分配，

Weibull:$G(x|\omega,\theta) = 1 - \exp(-(x/\theta)^{\omega})$，其中$\omega = 1.5$，$\theta = 20$

(1) 假設採用 LDF method 計算賠款準備金，請計算 AY 2013~2015 年賠款準備金之過程標準差(process standard deviation)。**(2 分)**

(2) 請畫出 normalized residuals plotted against the increment age of loss emergence，並依據你畫出的圖形評論採用 Weibull 分布的合適性。**(2 分)**

【參考解答】

(1)

We need σ^2. We have:

n=6

p= 5; one for each row and one for each column (except the first one)

Check for truncation at twice the age of the triangle:

G(66)= 0.998

Very close to 1.000; thus no truncation is necessary.

G(6)= 0.152

G(18)= 0.574

G(30)= 0.841

Ultimate Losses:

2013:5000/0.841=5945

2014:4000/0.574=6969

2015:2800/0.152=18421

Reserve=(5945-5000)+(6969-4000)+(18421-2800)=19535

Expected Cumulative Loss: G(x)*Ult. loss

Accident Year	12 Months	24 Months	36 Months
2013	904	3,413	5,000
2014	1,059	4,000	
2015	2,800		

Expected Incremental Loss

Accident Year	12 Months	24 Months	36 Months
2013	904	2,509	1,587
2014	1,059	2,941	
2015	2,800		

Actual Incremental Loss

Accident Year	12 Months	24 Months	36 Months
2013	2,500	1,600	900
2014	2,600	1,400	
2015	2,800		

$$\sigma^2 = \frac{1}{6-5}\left[\frac{(2500-904)^2}{904} + \frac{(1600-2509)^2}{2509} + \cdots + \frac{(2800-2800)^2}{2800}\right] = 6495$$

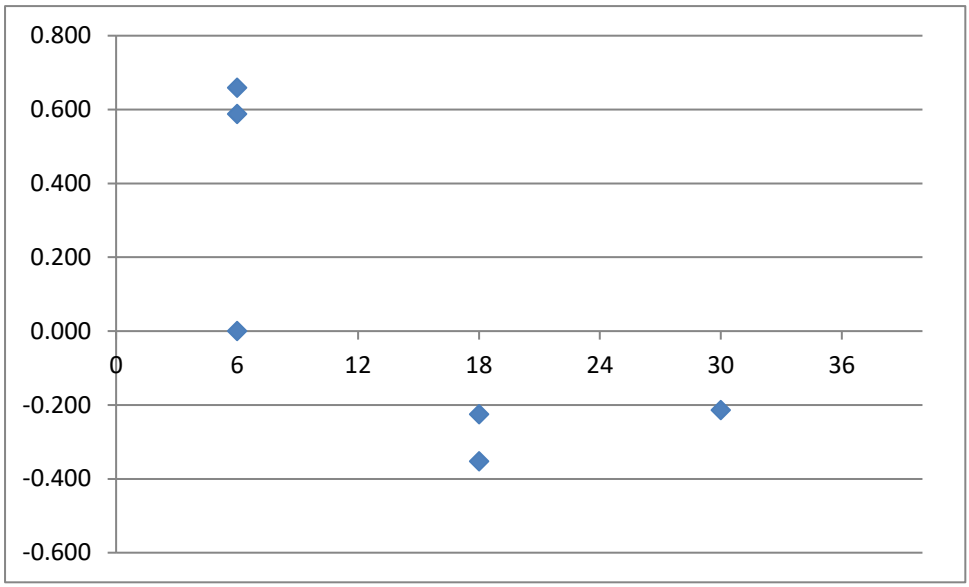
Process Standard Deviation: (6495*19535)^0.5=11264

(2)

Normalized Residuals

Accident Year	12 Months	24 Months	36 Months
2013	0.659	-0.225	-0.214
2014	0.587	-0.353	
2015	0.000		

Ex:0.659=(2500-904)/(6495*904^0.5)



This estimate of loss emergence is very poor. The residuals are positive for age 6, and negative for ages 18 and above. We expect a random fluctuation around zero, not this decreasing pattern.

15. **(3 分)**

某公司相關資訊如下：

- 投資人要求之Return on equity (ROE): 9%

單位:元

	2016年	2017年	2018年
Beginning GAAP Equity	120,000	125,000	132,000
Net Income	12,000	14,000	16,000
Minimum Capital Requirement	125,000	132,000	137,000

(3) 假設每年異常報酬成長率永久維持=3%，請計算該公司於 2016 年初之價值。**(1 分)**

(4) 假設每年異常報酬成長率於預測期間結束後 3 年=3%且 2022 年之後皆=0，請計算該公司於 2016 年初之價值。**(1 分)**

(5) 請評論上述 2 種情境假設何者較符合實際。**(1 分)**

【參考解答】

(3)

AE2016 = 12,000 - 120,000 * 0.09 = 2,000

AE2017 = 14,000 - 125,000 * 0.09 = 2,750

AE2018 = 16,000 - 132,000 * 0.09 = 4,120

V0 = BV0 + ΣAE/(1+k)i + Terminal Value

= 120,000 + 2,000/1.09 + 2,750/1.09^2 + 4,120/1.09^3 + [4,120*1.03/(0.09-0.03)]/1.09^3

= 181,211

(4)

V0 = 120,000 + 2,000/1.09 + 2,750/1.09^2 + 4,120/1.09^3 + 4,120*1.03/1.09^4 + 4,120*1.03^2/1.09^5 + 4,120*1.03^3/1.09^6

= 135,128

(5)

第(2)項is more realistic since maintaining abnormal earnings in perpetuity is not realistic in practice.

16（4分）

依據下列2間台灣保險公司資料回答問題：

・A保險公司只承保任意汽車保險且投資標的為存續期間與負債相同之台灣政府公債。

・B保險公司只承保multi-line commercial insurance且投資標的包含多種固定收益證卷及股票市場。

請依據下列風險項目，評析兩家公司面臨之問題。

(1) Pricing Risk（1分）

(2) Claim Variability(1分)

(3) Market Risk(1分)

(4) Correlation(1分)

【参考解答】

(1)

A保險公司: This line of business tends to be competitive, putting strong pressure on lower prices. The pricing risk is that prices will be set to low, so at to hit volume targets.

B保險公司: These accounts are not homogenous, and thus it is more difficult to get the pricing right. Also, if some of the business is long-tailed, it could be years before the insurer realizes it is underpriced.

(2)

A保險公司: Not a major risk, since this book is a high frequency, moderate severity book. Some items that could affect claim variability are inflation, changes in law or judicial perspective.

B保險公司: These policies tend to be high limits' policies, and thus the insurer has significant claim variability risk. The lack of homogeneity among clients makes this worse.

(3)

A保險公司: Since the Assets are liability matched to the liabilities, much of the market risk is hedged. There is still a minor risk that the bonds don't pay, or possibly fall in value. The largest risk here is that the duration calculation was
not done correctly, and thus the ALM match is off

B保險公司: Company is invested in equities and corporate bonds, both of which have significant market risks, which could significantly impact the balance sheet in a short period of time.

(4)

A保險公司: Correlation between the business (auto) and the assets (Taiwan Bonds) here is negligible.

B保險公司: If the company has investments in any of it's clients, the client could have an event that reduces the value of those investments, at the same time as an insured loss. More likely though, a downturn in the economy would
impact the prices of both the bonds and the equities the insurer is holding. This same downturn may lead the client to require less insurance, reducing the premium of the insurer.

17.(4 分)

根據以下至 2015 年之資訊回答下列問題

意外年度	平準保費 (on-level earned premium)	Growth Function	已發生賠款
2013	20,000	79.50%	10,500
2014	18,000	60.50%	6,500
2015	17,000	15.00%	1,800
總數	55,000		18,800

針對預估未付賠款之係數標準差(parameter standard deviation)為 800

意外年度之預期損失發展的 growth function 為 loglogistic function

$G(x \mid \omega, \theta) = x^{\omega} / (x^{\omega} + \theta^{\omega})$ ， ω=1.956， θ=15.286，

x 為平均意外發生日至評估日之月數

損失增量之發展乃根據 over-dispersed Poisson 分配，其中 scaling factor σ^2 = 9

(1)(2.5 分)

請採用 Cape Cod 法，並以五年為 truncation point 預估以上所有意外年度之未付賠款。

(2)(1.5 分)

請計算(1)之未付賠款的標準差(standard deviation)。

【參考解答】

(1)

Growth function for truncation = $x^{\omega} / (x^{\omega} + \Theta^{\omega}) = 54^{1.956} / (54^{1.956} + 15.286^{1.956}) =$ 0.922

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9) = (1) * ELR * (8)
意外年度	平準保費 (on-level earned premium)	Age at 12/31/2015	Average Age	Growth Function	= (1) * (4)	已發生賠款	ELR = (6)/(5)	=0.922 - (4)	預估未付賠款
2013	20,000	36	30	79.50%	15,900	10,500		12.7%	1,628
2014	18,000	24	18	60.50%	10,890	6,500		31.7%	3,658
2015	17,000	12	6	15.00%	2,550	1,800		77.2%	8,412
總數	55,000				29,340	18,800	64.1%		13,698

(2) Process variance = Mean * σ ^2 = 13698 * 9 = 123,282

Total Variance = Process variance + Parameter variance = 123282+800^2 = 763,282

Standard deviation = (763,282)^(1/2) = 873.7

18.(3 分)

根據以下某再保險公司截至 12/31/2015 為止之資料回答下列問題：

Calendar/ Accident Year	Earned Risk Pure Premium	Adjusted Premium	Aggregate Reported Loss	Reported loss lag	Chain Ladder IBNR
2011	12,500	13,000	11,500	95%	500
2012	13,300	13,800	10,000	85%	1,500
2013	14,000	14,200	8,500	72%	3,100
2014	15,500	15,800	7,900	55%	4,500
2015	16,000	16,000	6,000	40%	7,000
總數	71,300	72,800	43,900		16,600

(1)請採用 Standard Buhlmann 方法計算所有意外年度之 IBNR。(1 分)

(2)請分別描述若再保人採用 Standard-Buhlmann 方法計算 IBNR 之優點與缺點。(1 分)

(3)請採用 credibility-weighted estimate 的方法結合 chain ladder 法和 Standard-Buhlmann 法計算 IBNR，其中 chain ladder 法的 credibility factor 為 0.4。(1 分)

【參考解答】

(1) SB ELR = 43900/(13000*95%+13800*85%+14200*72%+15800*55%+16000*40%) = 88.9%

SB IBNR = 88.9% * (13000*5%+13800*15%+14200*28%+15800*45%+16000*60%) = 20,808

(2) 優點：採用實際損失資料計算 ELR，而非如 BF 法採用主觀判斷。

缺點：須調整每年的保費以反映過去的費率變更。

(3) Credibility-weighted IBNR =

(0.4*95%*500+(1-0.4*95%)*578+0.4*85%*1500+(1-0.4*85%)*1840+0.4*72%*3100+(1-0.4*72%)*3535+0.4*55%*4500+(1-0.4*55%)*6321+0.4*40%*7000+(1-0.4*40%)*8534)

= 19,891

19.(3 分)

請根據至 12/31/2015 為止之資料，依照 Sahasrabuddhe 文章中所述之方法，以意外年度 2015 年的成本水準為基礎，計算已發生損失發展三角形的最新對角線值的 basic limit of loss。

	已發生賠款			
意外年度	12個月	24個月	36個月	48個月
2012	333,000	612,000	650,000	700,000
2013	314,000	570,000	600,000	
2014	352,000	640,000		
2015	365,000			

上方之損失發展三角形是根據 unlimited basis

在 unlimited basis 下，歷年之損失發展趨勢為每年 4%

Basic Limit = 50,000

Exponential distribution 可以滿足理賠損失情況下，對於意外年度 2015 年之損失發展期間產生以下之 unlimited claim size mean：

	12	24	36	48
Unlimited Claim Size Mean	36,000	57,000	64,000	72,000

Mean of exponential distribution: θ

Variance of exponential distribution: θ ^2

Limited mean of exponential distribution at limit K: $\theta(1-e^{(\frac{-k}{\theta})})$

【參考解答】

	Triangle of Trend			
意外年度	12個月	24個月	36個月	48個月
2012	1.000	1.040	1.082	1.125
2013	1.040	1.082	1.125	1.170
2014	1.082	1.125	1.170	1.217
2015	1.125	1.170	1.217	1.265

	Triangle of Trended θ			
意外年度	12個月	24個月	36個月	48個月
2012	32,004	50,673	56,896	64,008
2013	33,284	52,700	59,172	
2014	34,615	54,808		
2015	36,000	57,000	64,000	72,000

AY 2012 at 12 months = 36000*(1/1.125) = 32004

	Limited Expected Value for basic limit			
意外年度	12個月	24個月	36個月	48個月
2012	27,023	33,291	34,699	36,047

↓

= 36000*(1-e^(-50000/36000)) = 27023

AY Cumulative loss at basic limit cost level

2012 = 700,000*(36,047/64,008) = 394,215

2013 = 600,000*(34,699/59,172) = 351,845

2014 = 640,000*(33,291/54,808) = 388,743

2015 = 365,000*(27,023/36,000) = 273,983

20.(5 分)

根據以下資訊回答下列問題

	累積已付賠款 ($ 000)		
意外年度'	12	24	36
2012	2,750	4,250	5,100
2013	2,700	4,300	
2014	2,900		

意外年度預期之損失發展(growth function)為 Weibull function 之形式:

$G(x|\omega,\theta) = 1 - \exp(-(x/\theta)^{\omega})$ ， ω=1.5， θ=20

(1)(2 分)

請採用 LDF 法計算意外年度 2012 年到 2014 年賠款準備金之 Process Standard Deviation

(2)(3 分)

請針對損失發展年度畫出 normalized residuals，並根據此圖形說明採用 Weibull model 之適當性

【參考解答】

(1)

Process st dev = $\sqrt{R\sigma^2}$ $\sigma^2 = 1/(n-p)\sum(c-\mu)^2/\mu$ n=6 p=2+3=5

↑ ↑

G,W Ult 2010, Ult 2011, Ult 2012

G(6)=1-$e^{(-(6/20)^{\wedge}1.5)}$ = .1515

G(18) = .5742

G(30) = .8407

* Check if truncation is needed:

- Extend Δ out 3 years G(66) = .9975
- G(66) is reasonably close to 1 and so the function does not need to be truncated

AY	G(x)	LDF = 1/G(x)	Ult = Cum Pd×LDF	Reserve
2010	0.8407	1.1895	6,066.4	966.4
2011	0.5742	1.7416	7,488.7	3,188.7
2012	0.1515	6.6007	19,142	16,242
				20,397.1

	avg age:	(6)	(18)	(30)
expected		12	24	36
	10	919	2564.3	1616.7
	11	1134.5	3165.5	1995.7
	12	2900	8091.3	5101.3

$\mu = [G(y) - G(x)]$ Ult AY

1995.7 = (.8407-.5742)(7488.7)

actual

10	2750	1500	850
11	2700	1600	
12	2900		↖4300-2700

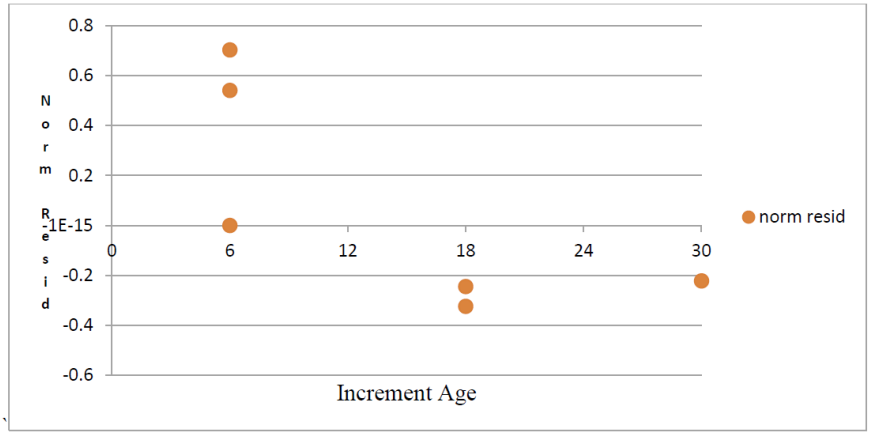
$\sigma^2 = 1/(6\text{-}5) \times \left[\frac{(2750-919)^2}{919} + \frac{(2700-1134.5)^2}{1134.5} + \ldots\right] = 7387.84$

process standard dev = $\sqrt{R\sigma^2} = \sqrt{7387.84 * 20397.1} = 12{,}275.607$

(2)

$\varepsilon = (c - \mu)/\sqrt{\mu\sigma\text{^}2}$

6	18	30
0.7027	-0.245	-0.222
0.5407	-0.324	↑
		(850-
0		1616.7)/(7387.4×1616.7)^0.5



若 Weibull function 是適當的，我們可以看到 residuals 應該接近 0，但在圖形中看到了隨者損失發展年齡增加，residual 由正數變為負數，因此 Weibull function 不不適當。

21.(4 分)

請解釋與說明五個技術上的困難，造成再保險公司準備金的預估比簽單保險公司困難。

【參考解答】

a. 再保險公司收到報案的時間比簽單公司更久（longer reporting lag）

b. 資料的同質性過低，無法用傳統的精算方法計算

c. 產業資料不實用

d. 再保險公司所收到的賠案資料不充分(例如無法區分意外年度)

e. 數據編碼(IT coding)與 IT 系統無法滿足再保險公司較複雜的需求

22.(4 分)

根據以下至 12/31/2014 為止的兩個比例型再保合約之資料：

合約	起保日	合約承保基礎	簽單保費	已發生賠款
No. 1	1/1/2012	損失發生時間 (loss occurring)	80,000	50,000
No. 2	7/1/2012	簽單時間 (policies written)	100,000	20,000

意外發展年齡 (月份)	最終損失發展因子
36	1.25
24	2.00
12	3.00

- 兩個比例型再保合約皆為一年期
- 再保險合約下之簽單保險合約(underlying policies)皆為一年期之事故發生制合約，合約之生效日平均分布在一年當中
- 再保合約 No.1 之佣金、仲介費與內部費用為保費之 5%
- 再保合約 No.2 之佣金、仲介費與內部費用為保費之 20%
- 損失發展因子可適用在該意外外年度的所有暴險(exposure)

(1)請採用 Standard Buhlmann 方法計算兩個再保合約結合後之最終賠款(1 分)

(2)具有長尾風險(long-tail exposure)特性時，若要計算 IBNR，何種情況下是 chian-ladder 方法勝過 Standard-Buhlmann 方法(1 分)

(3)具有長尾風險(long-tail exposure)特性時，若要計算 IBNR，何種情況下是 Standard-Buhlmann 方法勝過 chain-ladder 方法(1 分)

(4)請說明 Standard-Buhlmann 法相較於 Bornhuetter-Ferguson 法的主要創新(1 分)

【參考解答】

(1) 移除費用

Treaty 1 = 80000 * (1-5%) = 76000

Treaty 2 = 100000 * (1-20%) = 80000

在 12/31/2014, Treaty 1 是發展至 36 個月，Treaty 2 是發展至 24 個月(因平均簽單日是 1/1/2013)

Use-up premium = 76000 * (1/1.25) + 80000 * (1/2) = 100800

所有已發生賠款 = 50000 + 20000 = 70000

ELR = 70000/108000 = 0.694

IBNR = 0.694 * (76000 * (1-1/1.25) + 80000 * (1-1/2)) = 38308.8

最終賠款=IBNR+已發生賠款=38308.8+70000 = 108308.8

(2)將所有年度的保費調整到同樣的費率水準(rate level)

(3)當已發生賠款變動很大，但 ELR(預期損失率調整至同樣費率水準)是較穩定的

(4)ELR(預期損失率)是根據過去的損失經驗，而非武斷或經判斷選定的

23.(4 分)

藉由根據增量損失(incremental loss)建立 GLM(generalized linear model)，下方之損失

發展三角形將用於 bootstrap process，請回答下列問題：

	累積已發生賠款		
意外年度	12	24	36
2012	300,000	350,000	400,000
2013	400,000	450,000	
2014	550,000		

(1)請建立 log-link 損失發展三角形，以用於計算 GLM 中之參數(1 分)

(2)假設意外年度 2012 年於發展至 24 個月之累積已發生賠款為 200,000，而非 350,000，損失發展三角形中之其他值均不變，請解釋若要採用修改後之損失發展三角形建立 GLM bootstrap model 之困難(2 分)

(3)請根據(2)修改後之損失發展三角形建立 log-link 損失發展三角形，以用於計算 GLM 中之參數；並請簡述如何調整 GLM 的適配值(fitted values)(1 分)

【參考解答】

(1)

	增量賠款		
意外年度	12	24	36
2012	300,000	50,000	50,000
2013	400,000	50,000	
2014	550,000		
log-link triangle = ln (增量賠款)			
意外年度	12	24	36
2012	12.61	10.82	10.82
2013	12.90	10.82	
2014	13.22		

(2)

	增量賠款		
意外年度	12	24	36
2012	300,000	- 100,000	200,000
2013	400,000	50,000	
2014	550,000		

GLM bootstrap model 假設所有增量損失均不為負數

(3)

將(2)表格中所有值均加上 100,000

	增量賠款		
意外年度	12	24	36
2012	400,000	0	300,000
2013	500,000	150,000	
2014	650,000		
log-link triangle = ln (增量賠款)			
	增量賠款		
意外年度	12	24	36
2012	12.90	-	12.61
2013	13.12	11.92	
2014	13.38		

24.(4 分)

假設 A 保險公司有以下資訊:

Incremental Paid Losses

Accident Year	12 Months	24 Months	36 Months	48 Months	60 Months	72 Months
2010	1000	800	600	300	200	100
2011	1200	600	500	400	300	
2012	1000	1500	500	300		
2013	1300	1000	600			
2014	1500	1200				
2015	1400					

Accident Year	Earned Premium
2010	3000
2011	4000
2012	5000
2013	6000
2014	7000
2015	8000

請計算AY 2015年之Hurlimann’s Optimal Credibility Reserve。

【參考解答】

m_1=22.42%=(1000+1200+1000+1300+1500+1400)/(3000+4000+5000+6000+7000+8000)

m_2=20.40% , m_3=12.22% , m_4=8.33% , m_5=7.14% , m_6=3.33%

ELR=22.42%+20.40%+12.22%+8.33%+7.14%+3.33%=73.85%

p=22.42%/73.85%=0.304

q=1-p=0.696

z=0.304/(0.304+0.304^0.5)=0.355

R^{ind}=0.696*1400/0.304=3205

R^{coll}=0.696*(8000*0.7385)=4112

R^{C}=0.355*3205+(1-0.355*4112)=3790

25.(4 分)

某精算人員在進行2個業務險別巨災風險損失相關性評估，並取得下列經驗資料：

	Loss($000,000)	
Year	LoB 1	LoB 2
2009	42	35
2010	40	44
2011	38	34
2012	36	32
2013	34	30
2014	30	29

(1) 請詳細說明並計算 3 個統計測度(statistical measure)可用來測量上表中 2 個業務險別之相關性。(2 分)

(2) 請評估第(1)小題中之測度，何者較適用於測量此 2 個業務險別之相關性。(2 分)

【參考解答】

(1)

Kendall's τ - measures relative orders and does not take into account the magnitudes of values in the lines of business being compared. Weights are determined by the number of swaps.

	Loss($000,000)				
Year	LoB 1	LoB 2	Rank 1	Rank 2	Swap 1
2009	42	35	1	2	1
2010	40	44	2	1	2
2011	38	34	3	3	3
2012	36	32	4	4	4
2013	34	30	5	5	6
2014	30	28	6	6	5

N=6, Q=1

Kendall's $\tau = 1 - [4Q/N \times (N - 1)] = 1 - [4(1)/6 \times (6 - 1)] = 0.8667$

Spearman's rank correlation - measures relative orders and does not take into account the magnitudes of values in the lines of business being compared.

	Loss($000,000)

Year	LoB 1	LoB 2	Rank 1	Rank 2	Δrank	$(\Delta rank)^2$
2009	42	35	1	2	1	1
2010	40	44	2	1	1	1
2011	38	34	3	3	0	0
2012	36	32	4	4	0	0
2013	34	30	5	5	0	0
2014	30	29	6	6	0	0

N=6, S=2

Spearman's $\rho = 1 - {S}/{N(N^2-1)/6} = 1 - {2}/{6(6^2-1)/6}$ =0.9429

Pearson - a cardinal statistic weighting elements by the squares of their deviances from the mean.

Pearson ρ = cov(x, y)/[std(x)×std(y)] =0.7196

(2)

Ranks of preference for these measures is Kendall, Spearman, Pearson. Pearson is not well suited to measuring dependency in thick tailed, skewed distributions. Both Kendall's Tau and Spearman's rank are ordinal measures, but Kendall's Tau has greater dependence in the tails. As the portfolio is exposed to Cats it is more reasonable to use Kendall's Tau to describe dependence..

26.(4 分)

某保險公司 2014 年底相關資訊如下：

Gross Premiums : $100 million

Gross Loss Ratio: 55%

Gross Expense Ratio: 35%

Risk-free interest rate :2%

Investment Yield: 5%

Beginning Surplus: $500 million

Probability of distress without reinsurance :5%

該保險公司考慮購買下列再保險合約：

Proposed reinsurance :30% quota share

Ceding Commission 20% of reinsurance premiums

Probability of distress with reinsurance 1%

假設所有保費、費用及佣金皆在年初給付，所有賠款皆在年底給付，且業務規模未來年度皆無改變。

(1) 採用 risk-adjusted present value of future earnings，計算購買再保險前後保險公司價值的差異。(2 分)

(2) 除第(1)小題方法外，請說明另外 2 種方法可用來評估保險公司風險轉移的價值 (quantifying the value of risk transfer to the insurance company.)。(2 分)

【参考解答】

(1)

D=(1-d)/(1+r)

Without Reinsurance

D=(1-0.05)/(1+0.02)=0.9314

Premium	100	
Loss	55	=100*0.55
Expense	35	=100*0.35
Income	10	=100-55-35

Equity	500	
Premium	100	
Expense	35	
Investable Assets	565	=500+100+35
Investment Income:	28.25	=565*0.05

K=1/D-1=1/0.9314-1=7.37%

PV=(10+28.25)/7.37%=519

With Reinsurance

D=(1-0.01)/(1+0.02)=0.9706

Gross Premium	100	
Net Premium	70	=100*0.7
Loss	38.5	=70*0.55
Expense	35	=100*0.35
Commission	6	=100*0.3*0.2
Income	2.5	=70-38.5-35+6

Equity	500	
Net Premium	70	
Expense	35	

Commission	6	
Investable Assets	541	=500+70+35+6
Investment Income:	27.05	=541*0.05

K=1/D-1=1/0.706-1=3.03%
PV=(2.5+27.05)/3.03%=975

value of the firm increases=975-519=456

(2)
Sample Solution 1
(a) Efficient Frontier Graph - which plot U/W profit against different risk measure such as, VaR, or TvaR. The reinsurance option in the upper left corner is preferred since it has higher U/W profit with low risk
(b) Cost Allocation Method - holding capital is not free, which required a return of capital. We compare the total cost of holding capital and reinsurance net cost, to see which option has lower cost

Sample Solution 2
(a) Simple Factor: Value = prob of distress × Equity
Difference with or without reinsurance quantifies the risk transfer
(b) Efficient Frontier Graph: the expected U/W profit against a risk measure. More U/W profit expected should be associated with a higher risk measures. The closest to the efficient frontier of the program is the more efficient we are