G6 高等費率釐訂

Q1. (5 分)（中）

某汽車保險公司有2種銷售通路，分別是透過保險經紀人及網路銷售，而且費率結構中已考量兩個通路費用之差異，然公司行銷長要求精算師針對兩通路給予不同之純保險費。

精算師正在思考是否符合American Academy of Actuaries ”Risk Classification Statement of Principles.”

(1)請舉出並說明 2 項考量支持將通路差異納入純保險費費率因子中。(2.5 分)

(2)請舉出並說明 2 項反對支持將通路差異納入純保險費費率因子中。(2.5 分)

【參考解答】

(1)

Sample Responses to considerations supporting inclusion

• Statistical critieria: Credibility -> each classification is likely large enough to produce credible statistical predictions. I.e., enough policies sold through each distribution channel.

• Absence of Ambiguity -> each classification is easy to determine and likely to be mutually exclusive.

• Using distribution channels improve prediction accuracy of the expected loss of the insured.

• Since more policies can be priced more accurately, availability of coverage will increase.

• The consumer has a choice to either go to an independent agent and thus can control this selection.

• The distribution channel is easily measured and objective such that it is either one or the other.

(2)

Sample Responses to considerations against inclusion

• Manipulation -> easily manipulated by insured (Change distribution channel based on what produces preferential pricing)

• Public Acceptability -> unclear how distribution channel is related to the insured’s loss potential. No clear cause and effect relationship, not clearly based on relevant data.

• Hazard Reduction Incentive – Varying rates by distribution channel in no way promotes insureds to mitigate their hazard exposure because distribution channel is

not directly linked to losses.

• Using distribution channels is more prone to insured’s manipulation. They can price through different channels and select the lowest price. • It is hard to justify for the causality to the DOI regulator to make the variable acceptable.

• Distribution channel does not necessarily reflect differences in expected loss. No reason to believe driving behavior is different and so causality does not appear to be here.

• A consumer one year could go to an agent and then the next year go online so not constancy in measure.

【題目出處】

AAA : Risk Classification Statement of Principle

Q2. (6 分)(中)
GLM 模型較 univariate analysis 主要優點之一是 able to handle exposure correlation,然而, GLM 模型當處理的 predictor variables are very highly correlated 也會產生問題。

(1)Briefly state what can happen in GLMs with highly correlated predictor variables.
(2)Suggest two options for dealing with highly correlated predictor variables. Give a downside of each approach.
(3)Define multicollinearity, and state why it is a problem in GLMs.
(4)Briefly state one way to detect multicollinearity in a model.
(5)Define aliasing.
(6)Briefly state how GLM software can be used to correct for aliasing in a GLM.

【參考解答】
(1)
This can result in an unstable model with erratic coefficients that have high standard errors.
(2)
i. Removing all highly correlated variables except one. This eliminates the high correlation in the model, but it also potentially loses some unique information contained in the eliminated variables.
ii. Use dimensionality-reduction techniques such as principle components analysis or factor analysis to create a new subset of variables from the correlated variables, and use this subset of variables in the GLM. The downside is the additional time required to do this extra analysis.
(3)
Multicollinearity occurs when there is a near-perfect linear dependency among 3 or more predictor variables. For example, suppose x1 + x2 _ x3. When multicollinearity is present in a model, the model may become unstable with erratic coefficients, and it may not converge to a solution.
(4)
Use the variance inflation factor (VIF) statistic, which is given for each predictor variable, and measures the impact on the squared standard error for that variable due to collinearity with other

predictor variables by seeing how well other predictor variables can predict the variable in question.
(5)
When there is a perfect linear dependency among predictor variables, those variables are aliased.
(6)
Most GLM software will detect aliasing and automatically remove one of the problematic variables fromthe model.

【題目出處】
Goldburd : Generalized Linear Models for Insurance Rating

Q3 （4 分）（易）
請敘述可用來測試whether risk parameters shift over time的2個方法及步驟。

【參考解答】

Method 1: Chi-Squared Test
Null Hypothesis - H0: risk parameters do not shift over time
_ Group data into appropriate intervals
_ Calculate the overall expected value
_ Then calculate for each interval, $X^2 = (A-E)^2/E$;
Where A = actual observation and E = expected observation
_ Sum up X^2 for all intervals
_ If the total X^2 is > X^2 value from the X^2 table (based on degrees of freedom), then reject the null hypothesis that parameters do not shift over time. Therefore, accept the alternative hypothesis that risk parameters shift over time
_ If total X^2 < X^2 value based on degrees of freedom, then accept null hypothesis

Method 2: Correlation Test
_ Group data by pairs based on time lag
_ Calculate correlation for each pair
_ Calculate the average correlation by time lag
_ If the correlation decreases as time lag increases, then risk parameters shift over time

【題目出處】
Mahler :An Example of Credibility and Shifting Risk Parameters

Q4（6 分）**（難）**

假設某精算人員採用weighted k-means algorithm 進行 cluster analysis 進而產生new hazard groups for Workers' Compensation classes。且只採用限額$1,000,000之資料如下表所示:

Class	Premium	Cred-Wtd Excess Ratio	Current hazard group
1	150,000	0.455	A
2	200,000	0.355	B
3	150,000	0.281	A
4	250,000	0.555	C
5	400,000	0.361	A
6	350,000	0.581	C
7	200,000	0.204	B

精算人員決定與現行相同採用3個hazard groups.請計算經過weighted k-means algorithm一次疊代(iteration)後每一個Class會位於哪一個hazard groups 。

【參考解答】

compute the centroid of each current hazard group:

$\overline{R_A}$=(150,000*0.455+150,000*0.281+400,000*0.361)/(150,000+150,000+400,000)=0.364

$\overline{R_B}$=0.280

$\overline{R_C}$=0.570

Next, compute the distances to each centroid by subtracting the class excess ratio from the centroid excess ratios, and assign each class to the hazard group with the closest centroid:

Class	Dist to A	Dist to B	Dist to C	New Hazard Group
1	0.091	0.176	0.115	A
2	0.009	0.076	0.215	A
3	0.083	0.002	0.289	B
4	0.191	0.276	0.015	C
5	0.003	0.082	0.209	A
6	0.217	0.302	0.011	C
7	0.160	0.076	0.366	B

【題目出處】

Robertson :NCCI’s 2007 Hazard Group Mapping

Q5:(4.5 分)（易）

令 X 等於損失金額，下表為 X 在不同區間的平均數：

Size of Loss Range 損失金額範圍	Average Size of Loss 損失金額平均數
X <= $1,000	$500
$1,000 < X <= $2,000	$1,500
X > $2,000	$4,000

另有以下資訊：

總賠案數	500
損失金額平均數	$1,300
安全係數	0.80
自負額$1,000 的 loss elimination ratio Tempered loss elimination ratio for a $1,000 straight deductible	0.40

Calculate the tempered loss elimination ratio for a straight deductible of $2,000.

請計算自負額$2,000 的 loss elimination ratio

【參考解答】

令損失金額小於$1,000 的賠案數為 A

$$0.4=0.8\times\frac{500A+1000(500-A)}{500\times1300}$$

求出 A=350

令損失金額介於 1,000 到$2,000 的賠案數為 B

$$1300=\frac{500\times350+1500B+4000(500-350-B)}{500}$$

求出 B=50

損失金額超過 $1,000 的賠案數為 500 - 350 - 50 = 100

$$\text{Tempered PLER at \$1,000}=0.80\times\frac{(500)(350)+(1500)(50)+(2,000)(100)}{500\times1300}=0.55$$

【題目出處】

Gillam and Snader 的”Fundamentals of Individual Risk Rating”

Q6：(4.5 分)（中）

某精算師正計算著小機械製造商的勞工補償保險，其保單生效日為 2015/7/1 以下資料乃是該公司參加的回朔費率計劃

- 損失為 $0 到$1,000,000 的 uniform distribution
  Its losses are expected to be uniformly distributed between $0 and $1,000,000.
- 最高保費時的損失為$700,000 (Losses at maximum premium are $700,000.)
- 最高保費時的損失為$200,000 (Losses at minimum premium are $200,000.)
- 損失轉換因子 1.20 (The loss conversion factor is 1.10.)
- 基本保費 $125,000 (The basic premium is $100,000.)
- 稅賦因子 1.00

a. (2 分)
Calculate the expected retrospective premium for this workers compensation policy.
請計算該保單的預期保費

b. (2.5 分)
The company is considering whether to implement a fraud detection device. This addition would result in a shift in the loss distribution to a uniform distribution between $0 and $800,000.
Assuming that no other plan parameters change, calculate the resulting premium savings.
該公司考量加入一個反詐欺偵測系統，可以使損失降低為 $0 到 $800,000 的 uniform distribution，假設其他參數不變，請計算預期減少的保費

【參考解答】

a.

$E[R] = (b + C(E[A]-I))T$

$E[A] = (0.5)(\$1,000,000 - \$0) = \$500,000$

$C = 1.20$

$b = \$125,000$

$T = 1$

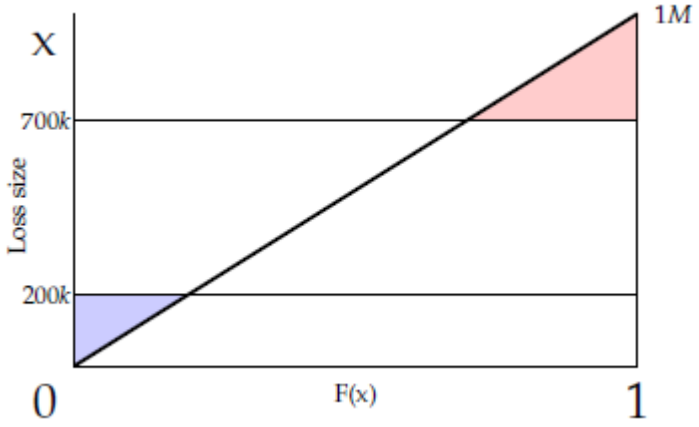
$I = (\Phi(r_G) - \Psi(r_H))E[A]$

$\Phi(r_G)E[A] = (0.5)(0.3)(\$300k) = \$45,000$

$\Psi(r_H)E[A] = (0.5)(0.2)(\$200k) = \$20,000$

$I = \$45,000 - \$20,000 = \$25,000$

$E[R] = (\$125,000 + 1.20(\$500,000 - \$25,000))*1.00 = \$695,000$



b.

$E[A] = (0.5)(\$800,000 - \$0) = \$400,000$

$I = (\Phi(r_G) - \Psi(r_H))E[A]$

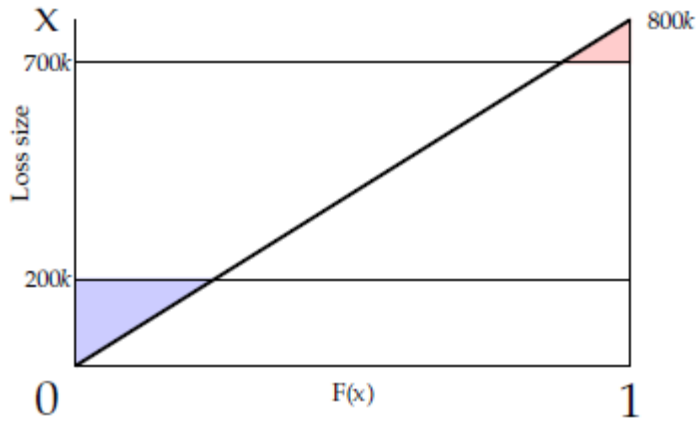
$\Phi(r_G)E[A] = (0.5)(0.125)(\$100k) = \$6,250$

$\Psi(r_H)E[A] = (0.5)(0.25)(\$200k) = \$25,000$

$I = \$6,250 - \$25,000 = -\$18,750$。

$E[R] = (\$125,000 + 1.20(\$400,000 - (-\$18,750)))*1.00 = \$627,500$

保費減少 $= \$695,000 - \$627,500 = \$67,500$



【題目出處】

Lee ” The Mathematics of Excess of Loss Coverages and Retrospective rating - A Graphical Approach”

Q7:(5 分)**(難)**

An actuary prices two loss-sensitive options for a workers compensation policy as follows:
關於勞工補償保險保單，現有兩種損失敏感的選擇
Option 1: A large deductible plan with a per-occurrence deductible of $50,000
選擇1:高自負額計畫，每事故自負額$50,000
Option 2: An incurred retrospective rating plan with the following parameters:
選擇2:回朔保費計劃

每事故限額Per Occurrence Limit	$50,000
基本保費Basic Premium	$150,000
稅賦因子Tax Multiplier	1.05
損失轉換因子Loss Conversion Factor	1.2
最初保費Deposit Premium (paid at policy inception)	$800,000

For each of the options above, assume that no aggregate limits or maximum premiums apply and that the first adjustment will take place 18 months after policy inception.
針對以上兩種選擇，假設第一次調整時無總損失限額或最高保費限額，且於保單生效後18個月調整。
Additionally, the actuary has developed the following assumptions for the insured:
另外還有以下假設：

	未設限額 Unlimited	限額$50,000 Limited to $50,000
預期損失 Expected Loss	$650,000	$435,000
18個月到最終已報賠款損失發展因子 18-Ultimate Incurred LDF	4.25	3.75
18個月到最終已付賠款損失發展因子 18-Ultimate Paid LDF	8.80	6.55

a. (3.5 分)
For each of the plans above, determine the expected cash flows

between the insured and insurer
18 months after policy inception.
以上兩種選擇，請求出保險公司與投保公司雙方在 18 個月調整時的預期現金流量

b. (1.5 分)
The insured is contemplating a third option of purchasing an excess policy with a self-insured
retention of $50,000.
投保公司正考慮第三種選擇，購買一超額保單，自負額為$50,000

- Which of the three options would be least attractive to the insurer if they wish to minimize credit risk? Briefly explain your choice.
  若考量降低信用風險，哪一種選擇是最不吸引保險公司的，請說明
- Which of the three options would be least attractive to the insurer if they wish to minimize interest rate risk? Briefly explain your choice.
  若考量降低利率風險，哪一種選擇是最不吸引人保險公司的，請說明

【參考解答】

a.

選擇1 高自負額計畫
第18個月(未設限額)的預期已付賠款= 650,000 / 8.80 = 73,864
第18個月(限額下)的預期已付賠款= 435,000 / 6.55 = 66,412
保險公司已付理賠$73,864
投保公司預期付給保險公司$66,412
選擇2 回朔費率計畫:
第18個月的預期保費= (b + CA* + PCF)T
A* = 435,000 / 3.75 = 116,000
PCF = 1.2 × (650,000 - 435,000) = 258,000
R*= (150,000 + 1.2×116,000 + 258,000) × 1.05= 574,560
第18個月的預期現流= 574,560 - 800,000 = -$225,440
投保公司預期從保險公司獲得退保費$225,440

b.

- The LDD policy would be least attractive as it has the highest credit risk since the insured may not be able to pay the insurer for amounts below the deductible. Excess policies have no credit risk since the insurer only pays losses in excess of

the retention, and retrospective policies have deposit premiums that are paid up-front that are usually equal to the expected guaranteed cost premium.

選擇1高自負額計畫會自最不吸引人的，因為有最高的信用風險，投保公司可能無法給保險公司低於自負額的金額。選擇3超額保單沒有任何信用風險，因為保險公司只要給付高於自留額的部分。選擇3回朔費率計畫會先收取最初保費，通常會與預期保證成本保費(expected guaranteed cost premium)，所以雖然有信用風險但是風險低於選擇2。

- An excess policy would be least attractive as it has the highest interest rate risk, since the insurer is paid premiums up front, and won't have to pay losses and expenses for many years. LDD and retro policies incur costs sooner since the insurer services the policies.

  選擇3超額保單是最不不吸引人的，因為有最高的利率風險，因為保險公司先收了保費以後，會在很多年之後才給付賠款與費用。選擇1與2都有在中途依據保單約定而有給付賠款或退保費的動作，所以利率風險較低。

【題目出處】

Teng "Pricing WC Large Deductible and Excess Insurance"

Q8:(4.5 分)（中）

依據 Gillam and Snader 的”Fundamentals of Individual Risk Rating”以及下列資料，回答以下問題：(假設所有的風險有相同的標準保費)(Show all the works)

風險數目	未設限額之損失率
1	10%
4	30%
2	40%
4	50%
1	60%
3	70%
1	80%
2	100%
2	120%

Number of Risks 風險數目	Limited Loss Ratio 設限額之損失率
1	10%
4	30%
4	40%
3	50%
2	60%
1	70%
2	80%
2	90%
1	110%

a. (0.5 分)

請計算未設限額的預期損失率與設限額的預期損失率

b. (0.5 分)

請計算 loss elimination ratio

c. (2.5 分)

請計算損失率 0%到 120%之間每 10%的 Table M charges

d. (1 分)

請計算 entry ratio 為 0.5 時的 Table M savings

【參考解答】

(a)未設限額的預期損失率 = [(1)(10%) + (4)(30%) + ... + (2)(120%)] / 20 = 60%

設限額的預期損失率 = [(1)(10%) + (4)(30%) + ... + (1)(110%)] / 20 = 54%

(b) LER = 1 - 54% / 60% = 10%

(c)

Loss ratio	Entry ratio	# of risks 風險數	Risks / (total risks) 風險數占全部風險數比例	% above	Losses in next layer	Charge
.00	.000	0	.00	1.00	.167	1.000
.10	.167	1	.05	.95	.158	.833
.20	.333	0	.00	.95	.158	.675
.30	.500	4	.20	.75	.125	.517
.40	.667	2	.10	.65	.108	.392
.50	.833	4	.20	.45	.075	.284
.60	1.00	1	.05	.40	.067	.209
.70	1.167	3	.15	.25	.042	.142
.80	1.333	1	.05	.20	.033	.100
.90	1.500	0	.00	.20	.033	.067
1.00	1.667	2	.10	.10	.017	.034
1.10	1.833	0	.00	.10	.017	.017
1.20	2.000	2	.10	.00	.000	.000

(d) $\Psi(0.5) = \Phi(0.5) + r - 1 = 0.517 + .50 - 1 = .017$

【題目出處】

Gillam and Snader ” Fundamentals of Individual Risk Rating”

Q9：(5 分)（中）

下列是5個個別風險的未設限額損失率與設限額損失率：

Risk # 編號	Unlimited Loss Ratio 未設限額損失率(%)	Limited Loss Ratio 設限額損失率(%)
1	30	15
2	45	45
3	45	45
4	90	60
5	90	90

(a)(3.5分)

Calculate Table L charges at loss ratios of 0% to 90% using increments of 15%.

請列出0%到90%之間每15%的Table L charges

(b)(0.5分)

請描述當損失設了限額後對insurance charge的影響

(c)(1.0分)

實務上，抽樣損失率也許會不等於期望損失率，當不相等的情況發生時，請簡述兩種方法來解決這個問題。

【參考解答】

a.

E[A]/P = (30% + 45% + 45% + 90% + 90%)/5 = 60%

E[A*]/P = (15% + 45% + 45% + 60% + 90%)/5 = 51%

LER = (60%-51%)/60% = 0.15

Limited Loss ratio	# lim risks	# lim risks above	% lim risks above	Partial charges	Table L charges
0%	0	5	100%	1.00	1.00
15%	1	4	80%	.706	.75
30%	0	4	80%	.471	.55
45%	2	2	40%	.235	.35
60%	1	1	20%	.1176	.25
75%	0	1	20%	.0588	.20

90%	1	0	0%	0	.15

b.

答案1

Insurance charge decreases when loss limit is introduced due to overlap between excess loss
pure prem factor and insurance charge (assuming insurance charge here is not Table L insurance charge, which includes the LER, and is therefore increased.)

Insurance charge會減少，因為當損失設了限額後，超額損失純保費因子與insurance charge會重複(假設這裡的Insurance charge不是Table L insurance charge，Table L insurance charge包含了LER，Table L insurance charge會增加)

答案2

Insurance charge is increased, since both pre occurrence limit and aggregate limit decrease the
ratable loss.

Insurance charge會增加，因為每事故限額與總損失限額使費率釐訂採用的損失減少。

c.

- Create Table L using the sample loss ratio.
  採用抽樣損失率製作Table L
- Use the expected loss ratio, but at end divide both r and ϕ(r) by ϕ(o).
  採用期望損失率，但是用ϕ(o)除r跟ϕ(r)

【題目出處】

Skurnick “The California Table L”

Q10:(7.5 分)**(難)**

某保戶購買了高自負額的勞工補償保險，將在每事故賠款達$500,000開始補償，每年補償限額為$2,800,000

以下為該保單的資訊：

- 未設限額的期望損失為$1,600,000
- 損失分組調整因子 1.45
- 州與風險分組差別因子 0.9

期望損失範圍表(Table of expected Loss Rages)

Expected loss group 期望損失分組	Range of value($) 金額範圍
20	1,500,001-1,700,000
19	1,700,001-2,000,000
18	2,000,001-2,500,000
17	2,500,001-,3000,000
16	3,000,001-3,500,000
15	3,500,001-4,000,000

Insurance Charge表

Ranges of Entry Ratios Entry Ratios範圍	Expected loss group 期望損失分組					
	20	19	18	17	16	15
1.25-1.50	.512	.463	.425	.390	.369	.352
1.51-1.75	.436	.389	.359	.333	.301	.289
1.76-2.00	.387	.323	.302	.277	.247	.211
2.01-2.25	.344	.271	.247	.221	.201	.183
2.26-2.50	.277	.276	.273	.265	.261	.257
2.51-2.75	.182	.241	.261	.254	.252	.247

後來發現未設限額的期望損失為$2,000,000，請計算在這錯誤下保險公司多付的賠款占期望損失的比率

【參考解答】

$1.45 = \frac{1+(1.6M)k}{1-(2M)k}$ 解出 k = 0.2

原本的預測:
預期超額損失Expected Excess Loss = (0.2)($1,600,000) = $320,000
預期限額損失Expected Limited Loss = $1,600,000 - $320,000 = $1,280,000
調整後的預期損失Adjusted Expected Loss = $1,600,000×0.90×1.45 = $2,088,000
從 $2,088,000 查表，屬於第18組
計算Entry ratio，Entry ratio for aggregate = $2,800,000 / $1,280,000 = 2.1875
從Entry ratio查表得出 insurance charge = 0.247
預期損失Expected loss for insurer = (0.247)($1,280,000) + $320,000 = $636,160
修正後:
預期超額損失Expected Excess Loss = (0.2)($2,000,000) = $400,000
預期限額損失Expected Limited Loss = $2,000,000 - $400,000 = $1,600,000
調整後的預期損失Adjusted Expected Loss = $2,000,000×0.90×1.45 = $2,610,000
從$2,610,000查表，屬於第17組
計算Entry ratio，Entry ratio for aggregate = $2,800,000 / $1,600,000 = 1.75
從Entry ratio查表得出insurance charge = 0.333
預期損失Expected loss for insurer = (0.333)($1,600,000) + $400,000 = $932,800
錯誤比率% Error = ($932,800-$636,160)/$ 932,800 = 31.8%

【題目出處】
Fisher “Pricing Aggregates on Deductible Policies,”

Q11 : (4 分)

依據 Miccolis, “On the Theory of Increased Limits and Excess of Loss Pricing”，且根據下列資料，

每次事故限額	E[g(x; k)]	$E[g(x; k)^2]$
$ 300,000	50,000	$4(10^9)$
500,000	77,000	$5(10^{10})$
1,000,000	129,000	$6(10^{11})$

- 基本限額為$300,000
- 損失頻率為 Poisson 分配
- $\lambda = 10^{-8}$

(1) 請計算各保額加計風險考量(risk loading)之高保額係數。

(2) 若損失頻率為 Binomial 分配，請說明各保額加計之風險考量會增加或減少。

答：

(1) $I(500,000) = (77000+10^{-8}\times5(10^{10})) / (50000+10^{-8}\times4(10^9)) = 1.55$
$I(1,000,000) = (129000+10^{-8}\times6(10^{11})) / (50000+10^{-8}\times4(10^9)) = 2.698$

(2) 若為 Binomial dist.,
$Var(y) = E(n) \cdot Var(g(x)) + Var(n) \cdot E(g(x))^2 < E(n) \cdot E(g(x)^2)$
故加計之風險考量會減少。

Q12 (5 分) (中)

依據 Miccolis, “On the Theory of Increased Limits and Excess of Loss Pricing”，且根據下列資料，

每次事故限額	E[g(x; k)]	$E[g(x; k)^2]$
$ 500,000	25,000	$6(10^9)$
1,000,000	35,000	$7(10^{10})$
3,000,000	45,000	$8(10^{11})$

- 基本保額為$500,000
- 損失頻率為 Poisson 分配
- $\lambda = 10^{-8}$

(3) 請計算各保額加計風險考量(risk loading)之高保額係數。(2.5 points)

(4) 若損失頻率為 Binomial 分配，請說明各保額加計之風險考量會增加 或減少。(2.5 points)

答：

(3) $I(1{,}000{,}000) = (35000+10^{-8}\times 7(10^{10})) / (25000+10^{-8}\times 6(10^9)) = 1.42$
$I(3{,}000{,}000) = (45000+10^{-8}\times 8(10^{11})) / (25000+10^{-8}\times 6(10^9)) = 2.11$

(4) 若為 Binomial dist.,
$Var(y) = E(n) \cdot Var(g(x)) + Var(n) \cdot E(g(x))^2 < E(n) \cdot E(g(x)^2)$
故加計之風險考量會減少。

【題目出處】

Miccolis, “On the Theory of Increased Limits and Excess of Loss Pricing”

Q13: (3 分) (easy)

Gillam and Snader 列舉了三個可信度應滿足的條件，請分別以數學公式及文字敘述說明此三個條件。

答：

(1) $0 \leqq Z \leqq 1$
Credibility should not be less than zero and not greater than unity

(2) $dZ/dE \geqq 0$
Credibility should not decreases as the size of risk increases

(3) $d(Z/E)/dE < 0$
As the size of risk increases, the percentage charge for a loss of a given size should decrease

Q14 (4 分)

進行費率係數分析時，常建議以 GLM 等多維度分析取代傳統單維度分析(ony-way analysis)。

請簡述說明採傳統單維度分析計算費率係數的二個主要缺點，並舉例說明之。

答：

- One-way analyses can be distorted by correlations between rating factors. For example, young drivers may in general drive older cars. A one-way analysis of age of car may show high claims experience for older cars, however this may result mainly from the fact that such older cars are in general driven more by high risk younger drivers.
- One-way analyses also do not consider interdependencies between factors in the way they affect claims experience. These interdependencies, or interactions, exist when the effect of one factor varies depending on the levels of another factor. For example, the pure premium differential between men and women may differ by levels of age.

Q15 : (6 分)　Easy

某產險公司汽車保險高保額係數如下 (金額單位：萬元)：

每人限額	每次事故限額			
	100	150	200	300
100	1.00	1.40	1.66	2.00
150		1.48	1.73	2.20
200			1.78	2.30
300				2.45

請評估該係數是否存在不一致性(inconsistencies)，並請詳列計算過程。

Ans:

1.

每人限額	每次事故限額			
	100	150	200	300
100	1.00	1.40	1.66	2.00
150		1.48	1.73	2.20
200			1.78	2.30
300				2.45

1. 計算每次事故限額 200 及 300 之邊際費率

	200	300
	0.0014	0.0040
	0.0010	0.0020
		0.0015

未存在不一致性。

2. 計算每人限額 100 及 150 之邊際費率

100	0.0080	0.0052	0.0034
150		0.0050	0.0047

未存在不一致性。

3. 計算相鄰每次事故限額之係數差異

150 & 200	200 & 300
0.26	0.34
0.25	0.47

0.52

0.26>0.25，0.34<0.47<0.52

不一致性存在於 150 及 200。

4. 計算相鄰每人限額之係數差異

100 & 150	0.08	0.07	0.20
150 & 200		0.05	0.10

0.08>0.07<0.20，0.05<0.10

不一致性存在於 100 及 150。

Q16 : (5 分)

某車體損失險目前無自負額之規定，現欲新增扣減式自負額 (straight deductible) 之選擇。請依據下列資訊，計算自負額為$30,000 時的費率占原費率的比率。

賠款金額級距	賠款件數	總賠款金額(千元)
0 – 10,000	1,628	10,500
10,000 – 30,000	3,056	71,000
30,000 – 50,000	1,015	43,500
50,000 – 100,000	878	73,500
> 100,000	268	45,550

項目	%	是否隨總保費變動
賠款(含可分配理賠費用)	65	否
可分配理賠費用	3	否
佣金	15	是
稅	3	是
一般費用	15	否
利潤	2	是

答：

$k = (L_r + (N-n)r) / L$
$= (10,500+71,000+(1,015+878+268)*30)/(10,500+\ldots+45,550)$
$= 0.60$

$D = k(E-a) / (1-A-T-p)$
$= 0.6*(.65-.03) / (1-.15-.03-.02)$
$= 0.465$

$Rate_{30K} = 1 - .465 = 0.535$

Q17: (5 分)

某產險公司國外子公司車險商品之損失幅度如下：

車種別	地區別	
	北部	南部
機車	1,050	950
自用小客車	5,680	4,980
自用小貨車	4,590	4,220

為分析損失幅度，請建立廣義線性模型 (Generalized Linear Model)之求解方程式，並以矩陣方式表達 (不需計算求解)。

答：

令 B1 表示機車，B2 表示自小客，B3 表示自小貨，B4 表示北部

$\underline{Y} = X \cdot \underline{\beta} + \underline{\varepsilon}$

The vector of responses = $\underline{Y} = \begin{bmatrix} 1050 \\ 950 \\ 5680 \\ 4980 \\ 4590 \\ 4220 \end{bmatrix}$

Design matrix = $X = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Vector of parameters = $\underline{\beta} = \begin{bmatrix} \beta 1 \\ \beta 2 \\ \beta 3 \\ \beta 4 \end{bmatrix}$

$\underline{\varepsilon}$ = Error term = $\begin{bmatrix} \varepsilon\,1 \\ \varepsilon\,2 \\ \varepsilon\,3 \\ \varepsilon\,4 \\ \varepsilon\,5 \\ \varepsilon\,6 \end{bmatrix}$

Q18:(5 分)（中）

下表為再保人對原保險人契約之累積損失分佈：

損失率區間	區間平均損失率	區間損失機率
0-50%	44%	6%
50%-75%	67%	64%
75%-100%	86%	22%
>100%	109%	8%

原保險人對於損失率介於 75%至 100%之間的損失將承受 80%

(1)請計算再保險人經過損失調整前的預期損失率。(2.5 分)

(2)請計算再保險人經過損失調整後的預期損失率。(2.5 分)

【參考解答】

(1) Expected Gross Loss Ratio=(6%)(44%)+(64%)(67%)+(22%)(86%)+(8%)(109%)= 73.16%

(2) Expected LR net of corridor=(6%)(44%)+(64%)(67%)+(22%)(77.2%)+(8%)(89%)= 69.624%

- 77.2%=75%+(20%)(86%-75%)
- 89.0%=75%+(20%)(100%-75%)+(109%-100%)

【題目出處】

Clark：Basics of Reinsurance Pricing

Q19:(2 分)(易)

依據「Catastrophe Modeling：A New Approach to Managing Risk」之內容，請列出天災模型的四個模組為何?

【參考解答】

1. Hazard module
2. Inventory module(Exposure module)
3. Vulnerability module
4. Loss module

【題目出處】

Grossi & Kunreuther： “Catastrophe modeling：A New Approach to Managing Risk”

Q20:(6 分)（**中**）

假設一年內可能發生之四起事件資訊如下，且各事件間彼此獨立，每一事件只會發生一次或不發生，試回答下列問題：

事件	機率	損失金額
1	0.10	50,000
2	0.03	100,000
3	0.02	150,000
4	0.07	125,000

(1)請計算年平均損失(Average Annual Loss)。(2 分)

(2)請說明發生超越機率(Occurrence Exceedance Probability ,OEP)之定義。(1 分)

(3)請計算每一損失檔級之發生超越機率(OEP)。(3 分)

【參考解答】

(1)年平均損失

(AAL)=0.1*50,000+0.03*100,000+0.02*150,000+0.07*125,000=19,750

(2)The OEP is the probability that at least one loss exceeds the specified loss amount.

(3)各損失檔級 OEP 如下表所示 ：

事件	機率	損失金額	OEP
3	0.02	150,000	0
4	0.07	125,000	0.02
2	0.03	100,000	0.0886
1	0.10	50,000	0.1159
None		0	0.2043

【題目出處】

Grossi & Kunreuther： “Catastrophe modeling：A New Approach to Managing Risk”

Q21 ：(3 分)（易）

給定下列超額再保合約條件(excess of loss reinsurance)：

Retention as a Percentage Of Insured Value	Percentage of Premium Applicable to Retained Risk
10	48
20	62
30	71
40	79
50	85
60	89
70	93
80	96
90	98
100	100

某一財產之保險價值(insured value)為$500,000，主保險人(primary insurer)收取保費$5,000。

依據上表計算 the reinsurance premium for a limit of $200,000 excess of $100,000。

【參考解答】

100/500=20%

(100+200)/500=60%

Premium for layer=$500,000*(89%-62%)= $1,350

【題目出處】

Clark：Basics of Reinsurance Pricing