G5 高等準備金，價值評估與企業風險管理

1.(5 分)

A保險公司損失資料如下:

Accident Year	Cumulative Loss Payments		
	12 Months	24 Months	36 Months
2016	5,000	8,000	10,000
2017	5,500	8,400	
2018	6,000		

假設所有年度暴露數與保費皆相同且36個月後無損失發展。

(1) 請分別採用 loss-ratio based payout factors 及 Benktander method 計算 2018 年底之準備金金額。

(2) 請針對 AY 2018 年計算 Benktander method 第五次疊代之準備金金額。

(3) 假設$\mathrm{Var}(U_i) = \mathrm{Var}(U_i^{BC})$，請採用 Hurlmann Method 在最佳可信度及最小變異數前提下，估計 AY 2018 年未付賠款金額。

【參考解答】

(1)

Avg. Paid:

0-12 months = (5,000 + 5,500 + 6,000)/3 = 5,500

12-24 months = (3,000 + 2,900)/2 = 2,950

24-36 months = 2,000

U0 = 5,500 + 2,950 + 2,000 = 10,450

P1 = 5,500/10,450 = 0.526; Q1 = 1 - 0.526 = 0.474

P2 = (5,500 + 2,950)/10,450 = 0.809; Q1 = 1 - 0.809 = 0.191

1st iteration ultimate losses

2017 = 10,450 × 0.191 + 8,400 = 10,400

2018 = 10,450 × 0.474 + 6,000 = 10,950

2nd iteration ultimate losses

2017 = 10,400 × 0.191 + 8,400 = 10,390

2018 = 10,950 × 0.474 + 6,000 = 11,187

Total estimated Benktander outstanding losses as of December 31, 2018 =

10,390 + 11,187 －8,400 －6,000 = 7,177

(2)

3rd iteration ultimate losses from part A 2018 = 11,187 × 0.474 + 6,000 = 11,299

4th iteration ultimate losses from part A 2018 = 11,299 × 0.474 + 6,000 = 11,352

5th iteration ultimate losses from part A 2018 = 11,352 × 0.474 + 6,000 = 11,377

Reserve = 5th iteration Ultimate minus paid = 11,377 – 6,000 = 5,377

(3)

$Z^* = p1 / (p1 + \sqrt{p1}) = 0.526 / (0.526 + \sqrt{0.526}) = 0.420$

$RC = Z^* \times Rind + (1 - Z^*) \times Rcoll = 0.42 \times 11,400 + (1 - 0.42) \times 11,950 = 11,139$

Reserve = Estimated Ultimate minus Paid = 11,139 – 6,000 = 5,139

【題目出處】

W. Hürlimann Credible Claims Reserve: Benktander, Neuhaus and Mack

2.(3分)

假設A保險公司有以下資訊:

已發生損失率(Incurred Loss Ratio)

Accident Year	12 Months	24 Months
2015	0.225	0.445
2016	0.425	0.655
2017	0.413	0.651
2018	0.255	

(1)使用Least Squares Method估計AY 2018年於24個月之損失率。

(2)假設A保險公司採用Link Ratio Method及Budgeted loss Ratio Method的平均數估計AY2018年於24個月之損失率，請依據第(1)小題計算結果評估此方法是否為最佳(optimal)方法。

【參考解答】

(1)

$\bar{x}$=0.354, $\bar{y}$=0.584, $\overline{x^2}$=0.134, $\overline{xy}$=0.216

$\hat{b}$=(0.216-0.354*0.584)/(0.134-0.354^2)=1.067

$\hat{a} = \bar{y} - \hat{b}\bar{x}$ =0.206

$\hat{y}$=0.206+1.067*0.255=0.478

(2)

最佳方法即為 Least Squares method. 而 LS method 係為 Link Ratio Method 及 Budgeted loss Ratio Method 之加權,權重分別為 b/c 及(1-b/c)。

依據第 1 小題計算結果:b=1.067,c=0.548/0.354=1.65,

Link Ratio Method 權重=1.067/1.65=0.647(不等於 0.5)

因此採用 Link Ratio Method 及 Budgeted loss Ratio Method 的平均數估計不是最佳方法。

3.（5 分）

下列損失發展三角形為損失限額500,000元之資料，請評估損失限額250,000元之12-Ult及24-Ult損失發展因子

Cum Incurred Losses(000)

Accident Year	12 Months	24 Months	36 Months
2016	30,000	66,000	80,000
2017	33,000	72,000	
2018	36,000		

相關假設條件如下:

AY 年度趨勢值=5%，CY年度趨勢值=0%，損失金額假設服從指數分配且AY2018年12、24及36發展月數平均損失金額分別為280,000、320,000及350,000元。

【参考解答】

	AY TREND
2016	1.000
2017	1.050
2018	1.103

CY TREND	12 Months	24 Months	36 Months
2016	1.000	1.000	1.000
2017	1.000	1.000	1.000
2018	1.000	1.000	1.000

TREND	12 Months	24 Months	36 Months
2016	1.000	1.000	1.000
2017	1.050	1.050	1.050
2018	1.103	1.103	1.103

Unlimit Mean	12 Months	24 Months	36 Months
2016	253,853	290,118	317,316
2017	266,546	304,624	333,182
2018	280,000	320,000	350,000

Eg. for AY 2017, age 12 Months: 266546=280000*1.1/1.21

L=500,000,B=250,000

LEV (L)	12 Months	24 Months	36 Months
2016	218,439	238,346	251,676
2017	225,704	245,614	
2018	233,050		

Eg. for AY 2017, age 12 Months: 225704=266546*[1-(e^(-500000/266546))]

LEV (B)	12 Months	24 Months	36 Months
2016			
2017			
2018	165,344	173,493	178,660

Eg. for AY 2018, age 12 Months: 165344=280000*[1-(e^(-250000/280000))]

Finally, we can calculate C'

C'	12 Months	24 Months	36 Months
2016	22,708	48,042	56,791
2017	24,175	50,858	
2018	25,541		

Eg. for AY 2017, age 12 Months: 24175=33000*165344/225704

Now, we calculate the LDFs:

LDF_{12} =(48042+50858)/(22708+24175)=2.110

LDF_{24} =56791/48042=1.182

LDFs to Ultimate

LDF_{12}=2.11*1.182=2.494

LDF_{24} =1.182

【題目出處】

Sahasrabuddhe : Claims Development by Layer

4.（3 分）

依據下列資料，請測試並說明相鄰之損失發展因子是否存在相關性。

Cumulative Paid Losses

Accident Year	12 Months	24 Months	36 Months	48 Months	60 Months	72 Months
2013	14,000	18,000	20,000	21,000	21,800	22,000
2014	13,000	20,000	21,800	22,000	23,500	
2015	12,000	18,000	20,500	22,500		
2016	14,500	19,000	22,000			
2017	15,000	22,000				
2018	16,000					

【参考解答】

LDF

	12-24	24-36	36-48	48-60	60-72
2013	1.286	1.111	1.050	1.038	1.009
2014	1.538	1.090	1.009	1.068	
2015	1.500	1.139	1.098		
2016	1.310	1.158			
2017	1.467				

12-24 & 24-36

12-24 LDF rank	24-36 LDF rank	Difference	Squared
4	3	1	1
1	4	3	9
2	2	0	0
3	1	2	4

S=1+9+0+4=14

$$T_k = 1 - \frac{s}{\left(n * (n^2 - 1)/_6\right)}$$

T_{24}=1-(14/(4*15)/6)=-0.4

24-36 & 36-48

24-36 LDF rank	36-48 LDF rank	Difference	Squared
2	2	0	0
3	3	0	0
1	1	0	0

S=0

T_{36}=1-(0/(3*8)/6)=1

36-48 & 48-60

36-48 LDF rank	48-60 LDF rank	Difference	Squared
2	1	1	1
1	2	1	1

S=1+1=2

T_{36}=1-(2/(2*3)/6)=-1

T=(-0.4*3+1*2+(-1)*1)/6=-0.033
VAR(T)=1/((6-2)*(6-3)/2)=0.167
we use a threshold of 50%, which is the percentile range [25%, 75%]. Thus, the confidence interval is
CI=(-0.67*0.167^0.5, 0.67*0.167^0.5)=(-0.274,0.274)
The test statistic T= -0.033 is within the confidence interval. Therefore, we do not reject the Null Hypothesis that the adjacent LDFs are uncorrelated.

【題目出處】
Mack(1994) : Variability of Chain Ladder Reserve Estimates

5.（5 分）

依據下列資料並採用 90%信賴區間，請測試並說明是否存在曆年度趨勢現象。

Cumulative Paid Losses

Accident Year	12 Months	24 Months	36 Months	48 Months	60 Months	72 Months
2013	12,000	20,000	24,000	25,500	26,800	27,000
2014	18,000	20,000	25,000	27,500	29,000	
2015	12,000	18,000	21,000	23,000		
2016	15,000	25,000	28,000			
2017	10,000	22,000				
2018	15,000					

【參考解答】

LDF

AY	12-24	24-36	36-48	48-60	60-72
2010	1.667	1.200	1.063	1.051	1.007
2011	1.111	1.250	1.100	1.055	
2012	1.500	1.167	1.095		
2013	1.667	1.120			
2014	2.200				

Rank columns, calculate z=min(S,L) for each diagonal

AY	12-24	24-36	36-48	48-60
2010	*	L	S	S
2011	S	L	L	L
2012	S	S	*	
2013	L	S		
2014	L			

(若將 2010 AY 12-24 視為 L 及 2013 AY 12-24 視為*且對應之計算結果皆正確亦給分)

Diagonal	n	m	c_n	$E[z_n]$	$Var[z_n]$	z
1	0	0				
2	2	0	0.5	0.5	0.25	1
3	3	1	0.75	0.75	0.188	1
4	4	1	0.75	1.25	0.438	2
5	3	1	0.75	0.75	0.188	1

Sum of $E[z_n]$=0.5+0.75+1.25+0.75=3.25

Sum of $Var[z_n]$=0.25+0.188+0.438+0.188=1.064

Z=1+1+2+1=5

The Confidence Interval is $3.25 \pm 1.645 * \sqrt{1.064}$=(1.55,4.95)

Since Z=5, and it is not within the confidence interval, there are calendar year effects in the triangle.

【題目出處】

Mack(1994) : Variability of Chain Ladder Reserve Estimates

6.（3 分）

假設 A 保險公司於 2018 年底相關如下：

Accident Year	On-Level Premiums	Cumulative Paid Loss	Fitted Paid Emergence Pattern
2015	2,500,000	1,500,000	75%
2016	2,800,000	1,000,000	50%
2017	2,600,000	450,000	25%
2018	2,800,000	150,000	8%

Parameter standard deviation=800,000

Process variance/mean scale parameter(δ^2)=50,000

(1) 請採用 Cape Cod method 計算 A 保險公司於 2018 年底之賠款準備金。**(5 分)**

(2) 請計算第(1)小題賠款準備金之過程標準差(process standard deviation)。**(1 分)**

(3) 請計算第(1)小題賠款準備金之總標準差(total standard deviation)及變異係數(coefficient of variation)。**(2 分)**

【參考解答】

(1)

Total Premium*G(x)

=2,500,000*0.75+2,800,000*0.5+2,600,000*0.25+2,800,000*0.08=4,149,000

Total Loss

=1,500,000+1,000,000+450,000+150,000=3,100,000

ELR=3,100,000/4,149,000=74.7%

Accident Year	Premium	Expected Ultimate	Expected Emergence	Reserve
2015	2,500,000	1,867,500	25%	466,875
2016	2,800,000	2,091,600	50%	1,045,800
2017	2,600,000	1,942,200	75%	1,456,650
2018	2,800,000	2,091,600	92%	1,924,272
TOTAL				4,893,597

Eg. 2018: 2,800,000*0.747=2,091,600; 2,091,600*0.92=1,924,272

(2)

Process Variance =50,000*4,893,597

Process Stdev=(50,000*4,893,597)^0.5=494,651

(3)

Total Var = Process Var + Param Var

Total Var =800,000^2+494,651^2

Total Stdev=(800,000^2+494,651^2)^0.5=940,574

CV=940,574/4,893,597=19.2%

【題目出處】

Clark : LDF Curve-Fitting and Stochastic Reserving

7. (2 分)

請依據下列資訊回答問題：

Portfolio	Portfolio Size ($000,000)	Length of Claim Run Off(in years)	Selected Coefficients of Variation(CoV)	
			Outstanding Claim Liability	Premium Liability
A	200	20	8%	y
B	800	20	x	10%
C	800	3	5%	4%

Event risk 不顯著且不考慮。

請分別選定 x 及 y 值，並依據 internally benchmarking independent risk 詳細說明其合理性。

【參考解答】

x:

Sample Answer 1

Since Portfolio B has a very long claim run-off time, the Premium Liability COV should be higher than the OCL COV. Moreover, Portfolio B is larger (in size) than Portfolio A, which is also having the same length of claim runoff years. Thus the OCL COV for A is larger than the OCL COV for B.

8% > x, 10% > x

In addition, OCL COV for B is longer than OCL COV for C, since they are the same size, and C has a much shorter runoff time than B

x > 5%

Select x = 6%

Sample Answer 2

8% - since the tail of claims matches A (C is a lot quicker, so lower CV), it would be an appropriate CV to account for the uncertainty.

y:

Sample Answer 1

PL COV(A) > PL COV(B) (since A is smaller than B, but with the same runoff period)

PL COV(A) > OCL COV(A) (more uncertainty for PL in long tail lines)

Y > 10%   select y = 12%

Sample Answer 2

A has smaller size & longer runoff length than C that y should definitely be higher than 5%. Smaller book + same runoff length than B , y should be higher than 10%

I choose y to be 11% because it is longer tailed & smaller sized

Sample Answer 3

Y = 10% because this matches portfolio B which has a similar claim runoff length. Premium liability is risk that premiums written will not cover losses, and these two appear to write similar length (likely liability) coverage.

【題目出處】

Marshall : A Framework for Assessing Risk Margins

8. （2 分）

請說明採用Over-Dispersed Poisson Bootstrap Model時常見的4種資料問題，並針對每一個問題提供一個解決方式。

【參考解答】

Sample Answer 1

1.Negative incremental value: limit incremental losses to zero

2.Missing values/incomplete data: estimate missing values using surrounding values

3.Heteroscedasticity: stratified sampling is accomplished by organizing the development periods by group with homogeneous variance within each group and them sampling with replacement only from the residuals in each group

4.Exposures that have changed dramatically over the years: modify data to get pure premiums and multiply the residuals by the exposures by year after the process variance step

Sample Answer 2

1.Non-zero sum of residuals: add a single constant to all residuals so that the sum of the adjusted residuals is zero

2.Outliers/extreme values: exclude outliers from the average age-to-age factors and
residual calculations, but re-sample the corresponding incremental when simulating triangles

3.Heteroecthesious data/misshapen data/partial year/interim evaluation dates: project future incremental values before applying model; need to annualize then de-annualize results

4.Lack of residuals to sample from/lack of extreme residuals: need to parameterize a distribution from which to sample from

【題目出處】

Shapland, M., “Using the ODP Bootstrap Model: A Practitioner’s Guide” CAS Monograph Series, Number 4.

9.（3 分）

A 保險公司有汽車保險 2018 事故年度賠款於 2018 年底資訊如下:

- AY 2018年已付賠款:2百萬元
- 2018年滿期保費:8百萬元
- 初始預期損失率:60%
- 12-24個月損失發展因子:2
- 12-最終損失發展因子:2.5

（1）請分別以 Chain ladder method, Bornhuetter-Ferguson method, Benktander method 計算 AY 2018 年最終損失金額。

（2）請計算 AY 2018 年於 2019 年之已付損失增量，使得 AY 2018 年於 2019 年底時，Benktander 最終賠款估計值比 Bornhuetter-Ferguson 最終賠款估計值多 120,000 元，假設所有損失發展選定值同第(1)小題。

【参考解答】

(1)

CL：2,000*2.5=5,000(K)

BF：2,000+(1-1/2.5)*8,000*0.6=4,880(K)

Benktander:

q=1-1/2.5=0.6

R_{GB}=0.6*4,880=2,928

U_{GB}=2,000+2,928=4,928(K)

(2)

Let x= paid loss in 2019

24-Ult LDF=2.5/2=1.25

U_{BF}=(2,000+x)+(1-1/1.25)*8,000*0.6=2,960+x

[this is the losses at age 24 + expected losses * %unreported]

$R_{GB}=q_k U_{BF}$

q_k=1=1/1.25=0.2

R_{GB}=0.2*(2,960+x)=592+0.2x

U_{GB}=(2,000+x)+(592+0.2x)=2,592+1.2x

$U_{GB}=U_{BF}$+120

2,592+1.2x =2,960+x +120

x=2,440(k)

【題目出處】

Mack(2000)Credible Claims Reserve: The Benktander Method

10.（3 分）

某公司相關資訊如下：

- Return on equity (ROE): 10%
- Book value growth rate: 5%
- Discount rate: 7%
- 假設以上數值未來皆持續維持相同。
- 2018 年底該公司 book value 為$1,000,000.

(1) 採用 price-to-book value ratio 計算該公司於 2018 年底之價值。

(2) 請依據上述 ROE、Book value growth rate 及 Discount rate 之數值，評估假設這些數值未來持續維持相同之可能性。

【参考解答】

(1)

P / BV = 1 + [(ROE - k) / (k - g)]

= 1 + [(0.10 – 0.07) / (0.07-0.05)]

= 2.5

BV=1,000,000 * 2.5 = 2,500,000

(2)

It is not likely that the company will continue to see abnormal earnings indefinitely since as you continue to earn profit, this will attract more suppliers and prices will drop to remain competitive. This would lower the return until no abnormal earnings exist and returns are as expected. (Note that abnormal earnings is meant to represent earnings more than you your discount rate, your expected return)

11.(5 分)

Best 公司 2019 年意外年度最終賠款分別採用 Chain Ladder、Bornhuetter-Ferguson 及 Benktander 三種方法估計最終賠款，金額分別為 750 百萬元、690 百萬元及 735 百萬元。請問該意外年度事前估計(*a priori* estimate)最終賠款是多少？並請詳列計算過程。

【參考解答】

$U_{GB} = (1-q_k)\ U_{CL} + q_k\ U_{BF}$

$= (1-q_k^2)\ U_{CL} + q_k^2\ U_0$

$735 = (1-q_k)\cdot 750 + q_k\cdot 690$

$q_k = 0.25$

$C_k = U_{CL}\ (1-q_k) = 750\cdot 0.75 = 562.2$

$735 = (1-0.25^2)\cdot 750 + 0.25^2\ U_0$

$U_0 = 510$

【題目出處】 Mack, T. "Credible Claims Reserve: The Benktander Method,"

12.(2 分)

請簡述 Benktander 法相較於 Chain Ladder 及 Bornhuetter-Ferguson 二種方法的主要優點及缺點各一項。

【參考解答】

主要優點：

The mean squared error(MSE) of the Benktander reserve was almost as small as the MSE of the optimal credibility in most situations.

主要缺點：

As different actuaries may select a different U_0, they may have materially different estimates of the unpaid losses.

【題目出處】 Mack, T. “Credible Claims Reserve: The Benktander Method,”

13.(5 分)

依據下列資訊：

金額單位：百萬元

意外年度 Accident Year	滿期純保費 Earned Risk Pure Premium	調整保費 Adjusted Premium	已付賠款 Paid Losses	未付賠款 Case Reserves	已報賠款延遲 Aggregate Reported Loss Lag
2015	35	39	25	3	95%
2016	37	40	28	5	85%
2017	40	43	18	9	75%
2018	42	46	8	15	60%
2019	48	48	5	10	40%

(1)請以 Stanard-Buhlmann 法計算未報賠款準備金。

(2)請依據 Patrik，說明 Stanard-Buhlmann 法相較於 Chain Ladder 及 Bornhuetter-Ferguson 二種方法的主要創新及主要問題各一項。

【參考解答】

(1)

ELR = (25+3+28+5+18+9+8+15+5+10) / (39·95%+40·85%+43·75%+46·60%+48·40%) = 83.94%

IBNR = 83.94%·(39·(1-95%)+40·(1-85%)+ 43·(1-75%)+ 46·(1-60%)+48·(1-40%)) = 55.32（百萬元）

(2) 主要創新：

It incorporates reported loss into the estimation of expected ELR.

主要問題：

It is not clear how to adjust ELR to a prior estimate for each year.

【題目出處】 Patrik, G.S., "Reinsurance,"

14.(2 分)

依據下列資訊：

金額單位：百萬元

保單年度	保費（百萬元）		已報賠款（百萬元）	
Policy Year	18 months	27 months	18 months	27 months
2016	315	320	230	255
2017	360	365	255	280
2018	405	420	300	335
2019	410	435	310	345

請計算第一次回溯調整 PDLD ratio (premium development to loss development ratio for the first retro adjustment)。

【參考解答】

保單年度	(1)保費	(2)已報賠款	(3) = (1) / (2)
Policy Year	0-27	0-18	PDLD ratio
2016	320	230	1.391
2017	365	255	1.431
2018	420	300	1.400
2019	435	310	1.403
Average			1.406
Wtd Avg			1.406

【題目出處】Teng, M.T.S.; and Perkins, M.E., "Estimating the Premium Asset on Retrospectively Rated Policies,"

15.(3 分)

請回答下列關於再保險的問題：

(1)請簡述 3 項再保險人賠款報案延遲(claim report lags)通常較原保險人長的原因。

(2)請簡述 2 項再保險人已報賠款之發展通常較原保險人多的原因

【參考解答】

(1)
Primary carrier may not see potential for a claim to be come large, and may not notify reinsurer promptly.
Longer reporting pipeline. The claim has to be processed by the primary carrier, go through the reinsurance accounting system, then be reported to the reinsurer, and enter his system before the claim is recognized.
Some mass tort claims have extreme delays in discovery and in recognizing liability.

(2)
Tendency for primary carrier to underreserve ALAE.
Economic and social inflation have a larger impact on losses in the excess layers then in the primary layers.

【題目出處】 Patrik, G.S., "Reinsurance,"

16.(3 分)

依據下列資訊：

金額單位：百萬元

意外年度	滿期保費	已付賠款
2015	5000	3350
2016	5200	3640
2017	5500	3795
2018	5800	4350
2019	6100	4880

發展月	損失發展因子
12-24	3.50
24-36	2.00
36-48	1.10
48-60	1.05
60-最終	1.00

預期損失率為 75%。

(1)請以 Bornhuetter-Ferguson 法計算賠款準備金。

(2)請以(1)計算結果為基礎，繼續計算 Benktander 法之賠款準備金。

【參考解答】

意外年度	(1) 滿期保費	(2) 已付賠款	(3) Age-to Ultimate LDF	(4)=1-1/(3) q_k	(5)=(1)*75%*(4) R_{BF}	(6)=(2)+(5) U_{BF}	(7)=(6)*(4) R_{GB}
2015	5000	3350	1.000	0.0000	0	3350	0
2016	5200	3640	1.050	0.0476	185.64	3825.64	182.10
2017	5500	3795	1.155	0.1342	553.58	4348.58	583.58
2018	5800	4350	2.310	0.5671	2466.89	6816.89	3865.86
2019	6100	4880	8.085	0.8763	4009.07	8889.07	7789.49
Total							12421.03

【題目出處】 Mack, T. "Credible Claims Reserve: The Benktander Method,"

17.(2 分)

依據 Siewert, "A Model for Reserving Workers Compensation High Deductibles"，除損失率法(loss ratio approach)外，尚有其他估計溢額賠款的方法。

(1)請列出其他二種方法。

(2)針對(1)列出的二種方法，請分別說明其相對於損失率法的優點及缺點各一項。

【參考解答】

(1) **Implied loss development**

Direct loss development

(2)

Implied loss development advantages:

1. *Provides an estimate of excess losses at early maturities even when excess losses have not emerged.*

2. *Development factors for limited losses are more stable than those determined for losses above the deductible.*

3. *Estimating deductible losses helps determine the asset represented by revenue collected from the application of a loss multiplier to future losses.*

Implied loss development disadvantage:

Does not explicitly recognize excess loss development

Direct loss development advantage

1. *Explicitly focuses on excess development.*

Direct loss development disadvantages

1. *Factors tend to be quite leverages and extremely volatile, making selection difficult.*

2. *If excess losses have not actually emerged at any particular stage of development, it is not possible to get an estimate of the required liability.*

【題目出處】Siewert, "A Model for Reserving Workers Compensation High Deductibles"

18.（3 分）

依據下列資訊：

Unlimited Reperted Losses　　　　　　　　　　　　　　金額單位：百萬元

意外年度	發展月			
	12	24	36	48
2016	300	780	1170	1404
2017	380	1064	1596	
2018	410	1230		
2019	450			

Excess Reperted Losses

意外年度	發展月			
	12	24	36	48
2016	6.4	32.0	64.0	96.0
2017	7.6	45.6	91.2	
2018	9.4	75.2		
2019	10.0			

為評估所有意外年度最終超額賠款(excess losses)，請問採用 Direct development approach 和 Implied development approach 二種方法計算結果的差額。

【參考解答】

Direct development approach:

12-24: (32+45.6+75.2)/(6.4+7.6+9.4)=6.530
24-36: (64.2+91.2)/(32.0+45.6)=2.000
36-48: 96.0/64.0=1.5

2016: 96.0
2017: 91.2*1.5=136.800
2018: 75.2*2.0*1.5=225.600
2019: 10.0*6.53*2.0*1.5=195.900
Total=654.300

Implied development approach:

Limited Reperted Losses 金額單位：百萬元

意外年度	發展月			
	12	24	36	48
2016	293.6	748.0	1106.0	1308.0
2017	372.4	1018.4	1504.8	
2018	400.6	1154.8		
2019	440.0			

12-24: (748.0+1018.4+1154.8)/(293.6+372.4+400.6)=2.739
24-36: (1106.0+1504.8)/(748.0+1018.4)=1.478
36-48: 1308.0/1106.0=1.183

2016: 1308.0
2017: 1504.8*1.183=1780.178
2018: 1154.8*1.478*1.183=2019.138
2019: 440.0*2.739*1.478*1.183=2107.191
Total=7214.507

Unlimited Reperted Losses

12-24: (780+1064+1230)/(300+380+410)=2.82
24-36: (1170+1596)/(780+1064)=1.5
36-48: 1404/1170=1.2

2016: 1404
2017: 1596*1.2=1915.2
2018: 1230*1.5*1.2=2214
2019: 450*2.82*1.5*1.2=2284.2
Total=7817.4

Excess Losses = 7817.4-7214.507 = 602.893

Difference = 654.3-602.893 = 51.407

【題目出處】Siewert, "A Model for Reserving Workers Compensation High Deductibles"

19.(6 分)

請說明 3 項應用 Over-dispersed Poisson bootstrap 模型時常會遇到的資料議題，並請提供您建議的調整或處理方式。

【参考解答】

Sample Answer 1 :

- Negative incremental value: limit incremental losses to zero
- Missing values/incomplete data: estimate missing values using surrounding values
- Heteroscedasticity: stratified sampling is accomplished by organizing the development periods by group with homogeneous variance within each group and them sampling with replacement only from the residuals in each group
- Exposures that have changed dramatically over the years: modify data to get pure premiums and multiply the residuals by the exposures by year after the process variance step

Sample Answer 2 :

- Non-zero sum of residuals: add a single constant to all residuals so that the sum of the adjusted residuals is zero
- Outliers/extreme values: exclude outliers from the average age-to-age factors and residual calculations, but re-sample the corresponding incremental when simulating triangles
- Heteroecthesious data/misshapen data/partial year/interim evaluation dates: project future incre mental values before applying model; need to annualize then de-annualize results
- Lack of residuals to sample from/lack of extreme residuals: need to parameterize a distribution f rom which to sample from

The following data issues/adjustments were also accepted:

- Negative incremental value: remove the row from the triangle if it is causing extreme results an d doesn't improve the parameterization of the model; use -

  ln(abs(q(w,d))) for q(w,d)<0, when total of all incremental values in a development column is positive; when column(s) sum to a negative value, add the absolute value of the largest negative (either among the sums of development columns or incremental values in the triangle) to every incremental value in the triangle, solve the GLM, and reduce each fitted incremental value by the largest negative; when simulating from a negative incremental value m(w,d) using the gamma distribution: use Gamma(abs(m(w,d)), phi × abs(m(w,d))) + 2 × m(w,d), since this will have mean m(w,d) while remaining skewed to the right; discussion in 4.1.1 contains other alternatives

- Missing values/incomplete data: modify loss development factors to exclude the missing value (in which case there will not be a corresponding residual for this missing value)
- Outliers/extreme values: remove them and deal with them like missing values; exclude outliers from the average age-to-age factors and residual calculations, but re-sample the corresponding incremental when simulating triangles
- Heteroscedasticity: sort development periods into groups with homogeneous variances, multiply each residual in each group by a hetero-adjustment factor (max(st.dev.(ri))/st.dev.(ri)), sample with replacement among all residuals, and divide each residual by the hetero-adjustment factor when the residuals are resampled

【題目出處】 Shapland, M.,“Using the ODP Bootstrap Model: A Practitioner’s Guide”

20.(5 分)

根據以下某再保險公司截至 12/31/2018 為止之資料回答下列問題：

Calendar/Accident Year	Earned Risk Pure Premium	Adjusted Premium	Aggregate Reported Loss	Reported Lag	Chain Ladder IBNR
2014	13,300	13,900	10,000	95%	500
2015	14,000	14,500	8,500	86%	1,500
2016	15,500	15,800	8,000	75%	3,000
2017	16,000	16,300	6,200	65%	4,500
2018	17,200	17,300	5,800	50%	7,000
總數	76,000	77,800	38,500		16,500

(1)請採用 Standard Buhlmann 方法計算所有意外年度之 IBNR。

(2)請分別描述若再保人採用 Standard-Buhlmann 方法計算 IBNR 之優點與缺點。

(3)請採用 credibility-weighted estimate 的方法結合 chain ladder 法和 Standard-Buhlmann 法計算 IBNR，其中 chain ladder 法的 credibility factor 為 0.8。

【參考解答】

(1) SB ELR = 38500/(13900*95%+14500*86%+15800*75%+16300*65%+17300*50%) = 67.82%

SB IBNR = 67.82% * (13900*5%+14500*14%+15800*25%+16300*35%+17300*50%) = 14,263

(2) 優點：採用實際損失資料計算 ELR，而非如 BF 法採用主觀判斷。

缺點：須調整每年的保費以反映過去的費率變更。

Calendar/Accident Year	Earned Risk Pure Premium	Adjusted Premium	Aggregate Reported Loss	Reported Lag	Chain Ladder IBNR
2014	13,300	13,900	10,000	95%	500
2015	14,000	14,500	8,500	86%	1,500
2016	15,500	15,800	8,000	75%	3,000
2017	16,000	16,300	6,200	65%	4,500
2018	17,200	17,300	5,800	50%	7,000
總數	76,000	77,800	38,500		16,500

(3) Credibility-weighted IBNR =(0.8*95%*500+(1-0.8*95%)*471+0.8*86%*1500+(1-0.8*86%)*1377+0.8*75%*3000+(1-0.8*75%)*2679+0.8*35%*4500+(1-0.8*35%)*3869+0.8*50%*7000+(1-0.8*50%)*5866)

= 15,19

【題目出處】

Patrik, G.S., "Reinsurance," Foundations of Casualty Actuarial Science (Fourth Edition), Casualty Actuarial Society, 2001, Chapter 7, pp. 434-464 (section on Reinsurance Loss Reserving)

21.(5 分)

根據以下至 2018 年之資訊回答下列問題

	(1)	[2]	(3)	(4)	(5)	(6)	(7)	(8)	[9] = (1) * ELR * [8]
意外年度	平準保費 (on-level earned premium)	Age at 12/31/2015	Average Age	Growth Function	= (1) * (4)	已發生賠款	ELR = (6)/(5)	=0.922 - (4)	預估未付賠款
2016	20,000	36	30	79.50%	15,900	10,500		12.7%	1,628
2017	18,000	24	18	60.50%	10,890	6,500		31.7%	3,658
2018	17,000	12	6	15.00%	2,550	1,800		77.2%	8,412
總數	55,000				29,340	18,800	64.1%		13,698

針對預估未付賠款之係數標準差(parameter standard deviation)為 800

意外年度之預期損失發展的 growth function 為 loglogistic function

$G(x \mid \omega, \theta) = x^{\omega} / (x^{\omega} + \theta^{\omega})$, ω=1.956, θ=15.286,

x 為平均意外發生日至評估日之月數

損失增量之發展乃根據 over-dispersed Poisson 分配，其中 scaling factor σ^2 = 9

(1)請採用 Cape Cod 法，並以五年為 truncation point 預估以上所有意外年度之未付賠款。

(2)請計算(1)之未付賠款的標準差(standard deviation)。

【參考解答】

(1)

Growth function for truncation = $x^{\omega} / (x^{\omega} + \Theta^{\omega}) = 54^{1.956} / (54^{1.956} + 15.286^{1.956})$ = 0.922

(2)Process variance = Mean * σˆ2 = 13698 * 9 = 123,282

Total Variance = Process variance + Parameter variance = 123282+800ˆ2 = 763,282

Standard deviation = (763,282)ˆ(1/2) = 873.7

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9) = (1) * ELR * (8)
意外年度	平準保費 (on-level earned premium)	Age at 12/31/2015	Average Age	Growth Function	= (1) * (4)	已發生賠款	ELR = (6)/(5)	=0.922 - (4)	預估未付賠款
2016	20,000	36	30	79.50%	15,900	10,500		12.7%	1,628
2017	18,000	24	18	60.50%	10,890	6,500		31.7%	3,658
2018	17,000	12	6	15.00%	2,550	1,800		77.2%	8,412
總數	55,000				29,340	18,800	64.1%		13,698

【題目出處】

Clark, D.R., "LDF Curve-Fitting and Stochastic Reserving: A Maximum Likelihood Approach,"

22.(3 分)

請根據以下保險公司之財務資訊，依 abnormal earnings valuation model 計算公司於 1/1/2017 之價值

	2017	2018	2019
期初股東權益 (Equity)	1,000,000	1,090,000	1,190,000
淨損益 (Net Income)	150,000	166,000	183,000
年底支付股利 (Dividend)	60,000	66,000	73,000

- 預估 equity market risk premium = 8%
- 無風險利率(risk-free rate) = 2%
- 保險公司之 beta（β）= 1.2
- 公司計畫維持目前之股利 payout ratio = 60%
- 採用 CAPM 決定風險調整後之折現率(risk-adjusted discount rate)
- 假設 abnormal earning 在以上預估期間之後會逐年平均下降至 2024 年時降為零

【參考解答】

Required return = 2% + 1.2 * 8% = 11.6%

Abnormal earning in 2017 = 150,000 – 11.6% * 1,000,000 = 34,000

Abnormal earning in 2018 = 166,000 – 11.6% * 1,090,000 = 39,560

Abnormal earning in 2019 = 183,000 – 11.6% * 1,190,000 = 44,960

	abnormal earnings		PV of abnormal earnings
年	預估期間	逐年遞減期間	
2017	34,000		=34000/(1.116)^1=30466
2018	39,560		=39560/(1.116)^2=31763
2019	44,960		=44960/(1.116)^3=32347
2020		= 44960 * 4/5 = 35,968	=35968/(1.116)^4=23188
2021		= 44960 * 3/5 = 26,976	=26967/(1.116)^5=15578
2022		= 44960 * 2/5 = 17,984	=17984/(1.116)^6=9309
2023		= 44960 * 1/5 = 8,992	=8992/(1.116)^7=4171
2024		0	0
總數			146,822
期初股東權益			1,000,000
公司價值			1,146,822

【題目出處】

Goldfarb, R. "P&C Insurance Company Valuation,"

23.(3 分)

依照 Sahasrabuddhe 文章中所述之方法，有以下 A 和 B 兩位精算師預估 Exponential claim size model parameters 以第五年的成本水準為基礎，

	發展期間				
Actuary	1	2	3	4	5
A	50,000	120,000	165,000	195,000	205,000
B	25,000	65,000	95,000	115,000	120,000

兩位精算師均以 claim size model 預估累積發展因子，並依損失發展趨勢與保額的改變做調整。並將預估之累積發展因子與用 weighted-average link ratio 與所有發展期間所計算出之未調整的累積發展因子進行比較。

(1)請說明哪一位精算師預估的累積損失發展因子偏離尚未調整之發展因子較多？請說明不同的 claim size parameters 對於模擬出的損失發展的那些影響造成這樣的結果。

(2)請說明還有哪兩個其他模型的選擇會造成預估的損失發展因子偏離實際未調整的損失發展因子更大。

【參考解答】

(1)精算師 A 計算出的結果偏離較大，來自於 claim size parameters 較大，因保險金額的限制對於較大的賠款有較大的影響。所以針對保額進行調整時，賠款愈大調整會愈大。

(2)＊ 假設損失發展超過五年

＊ 假設不同的 claim size model 的分配，例如 gamma 或 pareto

＊ 不同意外年度損失趨勢的假設。

【題目出處】

Sahasrabuddhe, R., "Claims Development by Layer: The Relationship between Claims Development Patterns, Trend and Claim Size Models"

24 (3 分)

請說明採用 non-parametric smoothing model 進行賠款 run-off pattern 預估較採用 parametric curves model 的兩個優點。

【參考解答】

(1) Moments beyond the first 2 moments can be estimated for the loss distribution so
it can produce full predictive distribution.

(2) Non-parametric models can handle negative loss development such as salvage and
subrogation. Many parametric models do not support negative loss development.

【題目出處】

Verrall, R.J., "Obtaining Predictive Distributions for Reserves Which Incorporate Expert Opinion," Variance, Vol. 1, Issue 1, 2007, Casualty Actuarial Society.

25 (2 分)

精算師在建立隨機 chain ladder model 時，考慮以下之分配：

＊ Over-dispersed Poisson
＊ Over-dispersed Negative Binomial
＊ Normal

實際 2016 年意外年度之損失發展至 12 個月= 60,000
以 chain ladder model 預估 2016 年意外年度之損失發展至 24 個月= 80,000

(1)根據以下之損失分配模型，計算 2016 年意外年度之損失發展至 24 個月之變異數 (variance)。

Over-dispersed Poisson model；ϕ=1.5
Over-dispersed Negative Binomial model；ϕ=1.25
Normally distributed model；ϕ=1.75

(2)精算師想採用與 chain ladder 法有顯見連結之模型，請說明以上三個模型哪個適用。

【參考解答】

(1)Over-dispersed Poisson model：80000*(1/1-1/1.5)*1.5=40,000

Over-dispersed Negative Binomial model：1.25*(1.5-1)*1.5*60000=56,250

Normally distributed model：1.75*60000=105,000

(2)＊ Negative Binomial：其公式與 chain ladder 較接近

＊ Normal：其持續性的機率分配支持(-∞,+∞)

【題目出處】

Shapland, M.; and Leong, J.W.K., "Bootstrap Modeling: Beyond the Basics,"

26.(3 分)

請根據以下追溯費率之保險合約之資訊，計算 Retrospective premium asset。

以下為三個 retrospective adjustment 相關數據：

Retro Adjustment Period	% Loss Emerged	Loss Capping Ratio
第一	78%	0.92
第二	25%	0.75
第三	8%	0.56

- Basic premium factor = 0.25
- Loss conversion factor = 1.3
- Tax multiplier = 1.05
- 預期 unlimited loss = 300,000
- 預期損失率 = 75%
- 目前帳上之保費= 350,000
- 假設沒有任何保險合約已進行第一次 retro adjustment

【參考解答】

Cumulative capping ratio = 78%*0.92+25%*0.75+8%*0.56 = 95%

Capped loss = 300,000*95% = 285,000

Standard premium = 300,000/0.75 = 400,000

Premium asset = (0.25*400,000+1.3*285,000)*1.05-350,000 = 144,025

【題目出處】

Teng, M.T.S.; and Perkins, M.E., "Estimating the Premium Asset on Retrospectively Rated Policies,"（中）

27.(2 分)

某保險公司正在建立公司的 ERM 計畫(Enterprise Risk Management Program)，期計畫具以下特性：

(1) 僅包含保險風險和財務風險

(2) ERM 模型排除了公司的摩托車車責險，因該險的理賠期間很短並業務佔比很小。

(3) ERM 模型僅考量對公司有負面影響之情境假設，因如果模擬結果優於預期對公司而言並非風險。

(4) 因公司同時有商業險種和個人險種之業務，這兩種業務分別由不同部門管理，並有不同的核保準則，因此公司須建立兩套 ERM 制度，以分別管理。

(5) ERM 計畫將定期檢視。

請各說明兩點此套 ERM 計畫之優點和缺點。

【參考解答】

優點：

(1) 整套 ERM 計畫著重公司主要的風險，使資源運用更有效率。

(2) 計畫定期檢視，使公司可以快速調整以因應新的風險。

缺點：

(1) 其他風險例如營運風險、策略風險也應加入考量，使整體 ERM 計畫更健全。

(2) 優於預期的情境假設也需考量，因完整的 ERM 計畫需協助公司尋找更佳機會。

(3) 商業險種和個人險種仍具有相關性，若建立兩個獨立的 ERM 計畫將低估極端情境發生時所產生之負面影響。

【題目出處】

International Actuarial Association, "A Global Framework for Insurer Solvency Assessment," a research report of the Insurer Solvency Assessment Working Party, 2004, Chapters 1, 2, 5, 7, 8, and 9; Appendices B, D, E, H, and I.

28.(3分)

請根據以下保險公司持有之債券回答下列問題。

	到期年	信評	信用風險之資本需求	現行債券發行者無法履約之機率
債券A	1	BBB	45,000	0.30%
債券B	3	BBB	157,000	0.30%

假設最佳風險風散下，兩債券之 correlation coefficients = 0.6

(1) 請說明三個與債券投資組合相關之信用風險。

(2) 假設兩債券來自同一個發行人，請計算來自以上債券投資組合所產生信用風險所需之資本需求。

【參考解答】

(1) ＊ Default risk: 可能面臨債券發行人倒閉於無法履約之風險。

＊ Downgrade risk: 可能因債券發行人本身的信用風險而使其發行之債券遭到降級，其市價將會下降。

＊ 集中性風險：僅投資於相同信評等級之債券，並且信評等級並不高。

(2) 債券 A 和 B 來自同一個發行人，因此相關性等於 1，資本需求=45,000+157,000=202,000

【題目出處】

International Actuarial Association, "A Global Framework for Insurer Solvency Assessment," a research report of the Insurer Solvency Assessment Working Party, 2004, Chapters 1, 2, 5, 7, 8, and 9; Appendices B, D, E, H, and I.

29. (6 分)

請根據至 12/31/2015 為止之資料，依照 Sahasrabuddhe 文章中所述之方法，以意外年度 2015 年的成本水準為基礎，計算已發生損失發展三角形的最新對角線值的 basic limit of loss。

	已發生賠款			
意外年度	12個月	24個月	36個月	48個月
2012	333,000	612,000	650,000	700,000
2013	314,000	570,000	600,000	
2014	352,000	640,000		
2015	365,000			

上方之損失發展三角形是根據 unlimited basis

在 unlimited basis 下，歷年之損失發展趨勢為每年 4%

Basic Limit = 50,000

Exponential distribution 可以滿足理賠損失情況下，對於意外年度 2015 年之損失發展期間產生以下之 unlimited claim size mean：

	12	24	36	48
Unlimited Claim Size Mean	36,000	57,000	64,000	72,000

Mean of exponential distribution: θ

Variance of exponential distribution: θ^2

Limited mean of exponential distribution at limit K: $\theta(1-e^{(\frac{-k}{\theta})})$

【參考解答】

	Triangle of Trend			
意外年度	12個月	24個月	36個月	48個月
2012	1.000	1.040	1.082	1.125
2013	1.040	1.082	1.125	1.170
2014	1.082	1.125	1.170	1.217
2015	1.125	1.170	1.217	1.265

	Triangle of Trended θ			
意外年度	12個月	24個月	36個月	48個月
2012	32,004	50,673	56,896	64,008
2013	33,284	52,700	59,172	
2014	34,615	54,808		
2015	36,000	57,000	64,000	72,000

AY 2012 at 12 months = 36000*(1/1.125) = 32004

	Limited Expected Value for basic limit			
意外年度	12個月	24個月	36個月	48個月
2012	27,023	33,291	34,699	36,047

↓

= 36000*(1-e^(-50000/36000)) = 27023

AY　Cumulative loss at basic limit cost level

2012　= 700,000*(36,047/64,008) = 394,215

2013　= 600,000*(34,699/59,172) = 351,845

2014　= 640,000*(33,291/54,808) = 388,743

2015　= 365,000*(27,023/36,000) = 273,983

【題目出處】

Sahasrabuddhe, R., "Claims Development by Layer: The Relationship between Claims Development Patterns, Trend and Claim Size Models"