G6 高等費率釐定

Q1.（4 分）

(a)某精算人員進行費率釐訂，並取得下列資料

Type of Vehicle	Earned Exposures	Number of Claims per year	Pure Premium
Type A	250,000	8,000	300
Type B	100,000	6,000	500

精算人員預計將Type A及Type B分成2類進行費率釐訂，請說明此做法是否具有統計上合理性(statistically sound)。(2分)

(b)某精算人員進行費率釐訂，並取得下列資料

Type of Vehicle	Earned Exposures	Number of Claims per year	Pure Premium
Type A	40,000	2,000	100
Type B	400	80	1,000

精算人員預計將 Type A 及 Type B 分成 2 類進行費率釐訂，請說明此做法是否具有統計上合理性(statistically sound)。(2 分)

【參考解答】

1. There would be homogeneity within each class. There are enough exposures in each to have statistical credibility. These are mutually exclusive classes that could not be manipulated by the insureds. There are significant differences in pure premium. Yes, assigning Type A and Type B to different classes would be appropriate.
2. Solution 1: Don’t split type A and B

   No, assigning Type A and Type B to different classes would not be statistically sound. Even though Type B has much higher pure premium than Type A, there are only 50 exposures for Type B, which is too small to derive statistical conclusions. The high cost of Type B may only be random loss fluctuation.

   Solution 2: Split type A and B

   Yes, split them into different classes. While Type B has very small volume, examining the credibility-weighted differences between the types would still bring value. Type B is significantly worse in pure premium, and some of this difference would remain after credibility-weighting.

【題目出處】AAA : Risk Classification Statement of Principles

Q2.（6 分）

某精算師使用 GLM 分析車險歷史資料:

（1）請針對下列因子納入 GLM 分析提出反對的意見: (3 分)

i. Limit of liability.

ii. Number of coverage changes during the current policy period.

iii. ZIP code of the garaging location of the automobile.

（2）精算師採用 log-link function and a Tweedie error distribution (1<p<2)配適純保險費模型，請說明 2 項反對將自負額納入此模型之理由。(1.5 分)

（3）若不將自負額納入 GLM 模型中直接評估，請提出決定自負額係數的方法並說明決定之自負額係數如何融入於 GLM 模型中。(1.5 分)

【參考解答】

（1）

Samp le Responses for [a]

• Including limit of liability in the GLM can lead to counterintuitive results such as lower relativity for higher limit due to correlation with other variables not included in the model.

• Including limit may give unexpected results like lower rate for more coverage due to adverse or favorable selection.

Sample Responses for [b]

• The information will not be available for new business since we are building a GLM for the prospective period.

• Number of coverage changes is likely to change from what it is in the current policy period and thereafter year by year.

Sample Responses for [c]

• Too many ZIP codes to include it in the GLM; using a spatial smoothing technique would be more appropriate and include the determined value for ZIP code as an offset term in the GLM.

• Sparse data creates credibility concerns and it will add too many degrees of freedom to the model.

• There are too many ZIP codes to be used in a GLM. Furthermore, aggregating them into groups will cause a great loss of information.

• Too many ZIP codes create too many parameters which will potentially lead to overfitting

(2)

•Deductibles should lower frequency (small losses below deductible not reported) but increase severity (since claims that do get reported are higher average cost). This violates the assumption for Tweedie that variables move frequency and severity in the same direction.

• Deductible factors may produce higher relativities at higher deductibles due to factors other than pure losses elimination:

1. Insureds at high loss potential and high premiums may elect high deductibles to reduce premium 2. Underwriters may force high deductibles on high risks

• Deductible factors are likely correlated with other factors outside of the model and may give non intuitive results like paying more for less coverage; for example because underwriters force high risk insureds to purchase higher deductibles.

(3)

•The deductible relativities can be calculated using a mix of experience and exposure rating and then included in the GLM model as an offset.

• Determine deductibles relativities by means loss elimination calculation with historical data [i.e., portion of loss not paid because of deductible E(x;d)/E(x)]. Include the relativities as an offset term in the GLM.

• Deductible relativities should be determined based purely of loss elimination, outside of the GLM model. Then they should be included as offset factors in the log-link function as +ln(relativity).

【題目出處】

Goldburd : Generalized Linear Models for Insurance Rating

Q3. （4 分）

下表資料係計算experience of a single private passenger car 之可信度，假設賠款件數服從Poisson distribution。

Group	Last Accident	Earned Car Years	Premium at Present B Rates	Number of Claims
A	3 or more	450,000	300,000,000	20,000
X	2	350,000	200,000,000	17,000
Y	1	50,000	100,000,000	10,000
B	0	X	35,000,000	10,000
Total		850,000+X	635,000,000	57,000

(1) 假設 credibility for an insured that has no claim-free years=0.15，請計算 X 值。(2.5 分)

(2) 計算 2 年以上無理賠之可信度(credibility for the group of risks that have been claim-free for two or more years) (1.5 分)

【參考解答】

(1)

Mod=(10,000/35,000,000)/(57,000/635,000,000)=3.183

Mod = ZR + (1 - Z)

3.183=0.15*R+0.85 , R=15.55

R = 1/(1 – e^m)

m=0.06646

m=57,000/(850,000+X)

X=7708

(2)

Mod=[(20,000+17,000)/(300,000,000+200,000,000)]/(57,000/635,000,000)=0.8244

Z=1-Mod=0.1756

【題目出處】

Bailey & Simon :An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car

Q4.（4.5 分）

請依據Robertson’s ”NCCI’s 2007 Hazard Group Mapping”一文中採用之proposed hazard groups ，詳細說明3項符合American Academy of Actuaries’ ” Risk Classification Statement of Principles.”之考量。

【參考解答】

New hazard groups reflect the 3 statistical considerations of the AAA

1. Credibility: assigning a credibility of $z = \min\left(\frac{n}{n+k} * 1.5, 1\right)$gives larger classes more weight to permit the calculation of more accurate predictors of excess ratios.
2. Homogeneity: clustering analysis using k-means algorithm is used to assign each class into a hazard group. k-means has the property to minimize within variance and maximize between variance so new hazard groups are homogeneous.
3. Predictive Stability: since current hazard groups were used as the complement of credibility, this provided stability in the class excess ratios, while still recognizing any changes that were credible.

Q5.（6 分）

某精算人員針對 excess of loss workers compensation policy 進行定價，資料如下:

- injury types: fatal, permanent total injury (PT), major permanent partial (Major), minor permanent partial (Minor), temporary total (TT), and medical-only (Med).
- A multi-dimensional credibility technique (predicted) was used to estimate the frequency for class 5160.
- Class 5160 is in hazard group F.
- The hazard group relativities for Major, Minor, TT, and Med will be used.
- The multi-dimensional credibility relativities for PT claims will be used.
- Class 5160 is in Quintile 4 for both Fatal and PT claims.

Hazard Group F						
	Fatal	PT	Major	Minor	TT	Med
Frequency Relativity to TT	0.005	0.006	0.1	0.3	1	3.5
Severity Relativity to TT	80	100	50	8	1	0.5
Loss Elimination Ratio at $200,000	35%	20%	60%	100%	100%	100%

TT Frequency per $100 payroll	0.0002
TT Severity for Hazard Group F	$8,000

Hazard Group F for Fatal Claims			
	Predicted	Raw Data	Holdout Sample
Quintile 1	0.86	0.8	0.91
Quintile 2	0.92	0.9	0.95
Quintile 3	1	1	1
Quintile 4	1.05	1.15	1.04
Quintile 5	1.15	1.25	1.09
Mean	0.86	0.8	0.91

Hazard Group F for PT Claims			
	Predicted	Raw Data	Holdout Sample
Quintile 1	0.85	0.8	0.9
Quintile 2	0.95	0.9	0.96
Quintile 3	1	1	1
Quintile 4	1.1	1.1	1.04
Quintile 5	1.15	1.2	1.08
Mean	1	1	1

(1) 請評估是否應採用multi-dimensional credibility relativities 計算 expected loss for fatal claims。(2分)

(2) 依據第(1)小題結果，請計算expected loss for an excess of $200,000 workers compensation policy with $20 million in payroll。(4分)

【參考解答】

(1)

We want to use either the Hazard Group, credibility technique, or the raw data for fatal claims, based on which has the lowest SSE (compared to the holdout sample).

SSEPredicted = (0.86 - 0.91)^2 + (0.92 - 0.95) ^2 + (1.00 - 1.00) ^2 + (1.05 - 1.04) ^2 + (1.15 - 1.09) ^2 = 0.007

SSEHazard = (1.00 - 0.91) ^2 + (1.00 - 0.95) ^2 + (1.00 - 1.00) ^2 + (1.00 - 1.04) ^2 + (1.00 - 1.09) ^2 = 0.02

SSERaw = (0.80 - 0.91) ^2 + (0.90 - 0.95) ^2 + (1.00 - 1.00) ^2 + (1.15 - 1.04) ^2 + (1.25 - 1.09) ^2 = 0.052

The Predicted relativities result in the lowest Sum of Squared Errors for fatal claims, so they should be used for multi-dimensional credibility relativities instead of the fatal claims or the raw data relativities.

(2)

I assume that the payroll is all for class 5160.

For Med, TT, and Minor, the contribution will be $0 since the Loss Elimination Ratios are 100%,

Major: [($20,000,000 / $100)*0.0002*0.1*1.00]* [$8,000*50]*(1 - 60%) = $640,000

Since PT and Fatal uses the multi-dimensional credibility

relativities, look up the value for predicted for Quintile 4
for PT claims to get a relativity of 1.1 relative to the hazard group frequency.
PT: [($20,000,000 / $100)*0.0002*0.006*1.1]* [$8,000*100]*(1 - 20%) = $168,960
for Fatal claims to get a relativity of 1.05 relative to the hazard group frequency.
Fatal: [($20,000,000 / $100)*0.0002*0.005*1.05]* [$8,000*80]*(1 - 35%) = $87,360
Total Expected Excess Loss = $0 + $0 + $0 + $630,000 + $168,960 + $87,360 = $896,320

【題目出處】
Couret & Venter : Using Multi-Dimensional Credibility to Estimate Class Frequency Vectors in Work Comp

Q6. (4.5 分)

估計個別分類(individual class)之超額比例(excess ratios)其中一個方法是multi-dimensional credibility technique。

請就下列3項統計考量說明此方法優於estimating excess ratios by hazard group:

i. Homogeneity

ii. Credibility

iii. Predictive Stability

【參考解答】

i. Using the multi-dimensional credibility technique will result in excess ratios by class, instead of excess ratios by hazard group (a group of classes). The risks within a class will be more homogeneous than the risks within a group of classes.

ii. The multi-dimensional credibility technique both improves and worsens credibility of excess ratio estimates, in different ways. Credibility is improved because excess ratios for each injury type are calculated using data from other correlated injury types, so more information and credibility goes into the estimates. Credibility is worsened because the same data is subdivided much more finely by class instead of by hazard group, so the sample size that each excess ratio is based off of is much smaller.

iii. The multi-dimensional credibility technique both improves and worsens predictive stability, in different ways. Predictive stability is improved because data from more common minor injury types is included and these claims are more stable from year-to-year than the less frequent major injury types. Predictive stability is worsened because class level data is used, and the claims for each class will be more volatile from year-to-year than the claims at the hazard group level.

【題目出處】

Couret & Venter : Using Multi-Dimensional Credibility to Estimate Class Frequency Vectors in Work Comp

Q7.（8 分）

精算師使用參數 1 <p <2 的 Tweedie 分配建構了一個純保費模型，以確定勞工補償保險(Workers' Compensation)的表定費率(manual rates)。該模型輸出了每 100 元的工資(payroll)的純保費。該模型考量的變數如下：

變數	定義或資料來源	P value
產業	營造業、製造業或其他產業	0.002
是否有返回工作計畫	是，否	0.008
員工年齡	平均年齡	0.003
員工資歷	平均被雇用年數	0.005
地點		0.080
員工士氣	基於每年的公司問卷結果	0.150

精算師決定在模型中使用以下變數：產業，員工資歷和返回工作計畫。

(a)(1.5 分)

請說明包括此三個變數：產業，員工資歷和返回工作計畫的統計和非統計考慮因素。

(b)(1.5 分)

請說明不包括四個變數：員工年齡，地點，和員工士氣的統計和非統計考慮因素。

(c)(1.5 分)

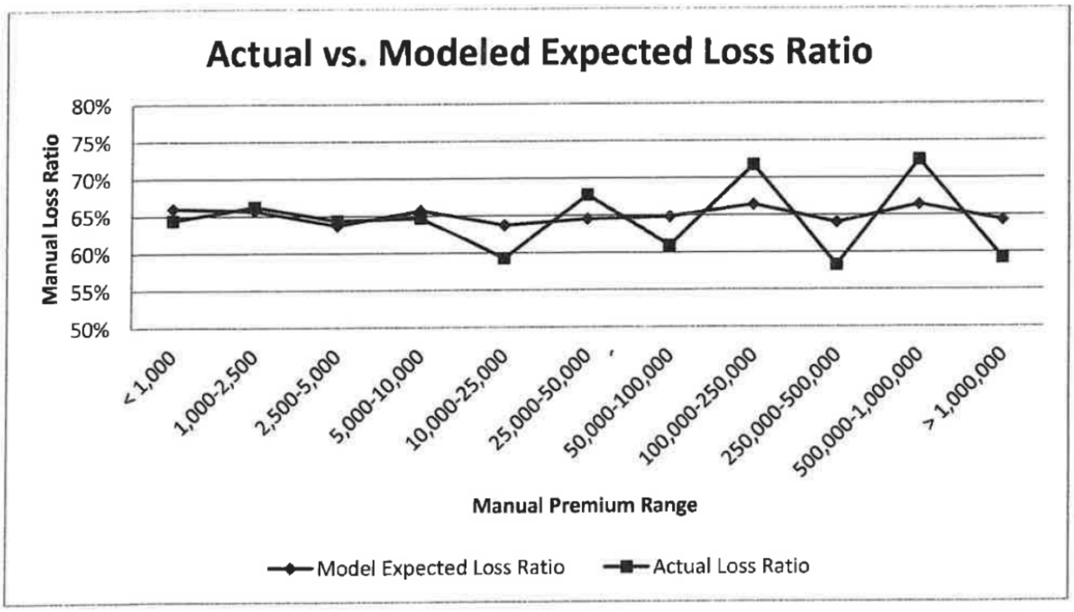
精算師使用這三個選定的變數來配適 log link GLM model 模型。在給定以下模型參數及製造業的勞工補償風險(Manufacturing Workers Compensation risk)，計算年度保單期間此風險的標準保費(standard premium)。

參數	Coefficient
Intercept 截距	-0.62
員工資歷	-0.04
是否有返回工作計畫：是	-0.2
產業別：營造業	0.35
產業別：其他	-0.55

製造業風險資訊	值
工資	1,000,000
員工資歷	5 年
是否有返回工作計畫	無
經驗費率法中的實際損失	12,500
固定費用	1,200
變動費用(保費的百分比)	15%
經驗費率法中常數(K)	10,000

(d)(1.5 分)

精算師依據不同的勞工補償保險風險規模(size of risk)，繪製了實際損失率與預期損失率的圖表，如下所示。為了改善模型對較大風險(larger risks)的配適程度，精算師正在考慮加入一個變數"最近 3 年歷史損失"進入模型。請說明三個反對的理由。



(e)(2 分)

精算師現在用此費率計畫，來評估一家工資 50,000,000 元的新成立營造業公司的風險(Construction risk)。

i. (1 分)

請說明在此費率計畫下為此風險制定保費的兩個潛在問題。

ii. (1 分)

為此風險提供另一種定價方法，並簡要說明這種替代方案對於被保險人以及保險公司的優點。

【參考解答】

(a)

產業別：p 值非常低，因此在 99.8%的水準上具有統計顯著性。它似乎在邏輯上

與預期損失相關、具有客觀性(objective)、實用性(practical)及可合法用於定價。

員工資歷：p 值非常低，因此在 99.5%的水平上具有統計顯著性。 員工的經驗似乎與預期損失在邏輯上相關，但從實用性(practical)的角度來看，需要使用公司的所有員工的數據，而且您必須考慮員工在一年內到職和離開公司，這將是一項挑戰。而且，它對新公司來說沒那麼有用。

返回工作計畫：p 值非常低，因此在 99.2%的水平上具有統計學意義。 這似乎與預期的賠償損失有因果關係，儘管可能難以核實，並且該計畫的成功可能因公司而有很大差異。

(b)

員工年齡：從統計角度來看，似乎應該包含此變數，因為它的 p 值非常低，為 0.003。但是，它可能與已包含的員工資歷變數高度相關，如果是這樣，那麼排除這個變數是有合理的。從實用性(practical)的角度來看，需要使用公司的所有員工的數據，而且您必須考慮員工在一年內到職和離開公司。

位置：從統計角度來看，這不是一個好變數，p 值為 0.08。然而，由於勞工補償福利和法規可能因地區而異，因此預期損失因地理區域而異，所以我仍然建議進行某種基於地區的定價。

員工士氣：這個 p 值高達 0.15，因此從統計角度來看，它不是一個好的變數。此外，士氣與預期損失的關係並不明顯，這些調查結果的有效性甚至可能會在因為蒐集方式受到質疑。所以我認為排除此變數是合理的。

(c)

PP rate 純保費率= exp(-0.62 - 0.04 × 5) = 0.4404

PP 純保費= 0.4404 × (1,000,000 / 100) = 4,404

Manual Prem 表定保費 = (4,404 + 1,200) / (1 - 15%) = 6,593

Mod = (12,500 + 10,000) / (4,404 × 3 + 10,000) = 0.97

Standard Prem 標準保費 = 6,593 × 0.97 = $6,395

(d)

- 以過去三年歷史損失當作變數，會與經驗費率法存在顯著重疊(overlap)，將此變數添加到模型中會導致重複計算損失經驗。
- 若使用歷史損失經驗為單一變數，不考慮風險大小(risk size)，隱含了將給予非常小的風險和非常大的風險相同的可信度，這並不合理。經驗費率法已經給更大風險(larger risks)更高的可信度。
- 無法保證添加歷史損失經驗可以提高對較大風險的配適度。它甚至可能導致我們過度配適訓練數據(training data)，這將使其在應用於未來風險時具有較低的預測性。

(e)

i.

•因為是新公司，沒有員工資歷的資料，可能也還沒有返回工作計畫，也沒有經驗費率法的經驗，因此只能根據產業別定價。

- 由於是較大風險(larger risks)，根據(d)部分的圖表，模型可能不太準確

ii.

建議採用回溯費率計畫(Retro rating plan)。回溯費率計畫的保險費乃基於被保險人在保單期內的實際損失。對保險公司來說，雖然一開始沒有太多數據可用於對此風險進行分類，但此方法最終能夠令保險公司更準確的計算保費。這也將使被保險人受益，因為如果他們有良好的經驗，他們最終會獲得較低的保費，這將使保險公司更有意願提供保障。

【題目出處】

Fisher “Individual Risk Rating”

Q8.（2.5 分）

給定以下平衡回溯費率計畫(balanced retrospective rating plan)的相關資料：

•最低保費的損失= $40,000

•最高保費的損失= $240,000

•損失轉換因子= 1.05

• e = $10,000

下表顯示某群具代表性的類似風險的實際經驗：

風險	實際總損失
1	20,000
2	40,000
3	80,000
4	80,000
5	120,000
6	140,000
7	160,000
8	240,000
9	280,000
10	440,000

(a)(2 分)請計算被保險人可以被收取的最高保費。

(b)(0.5 分)精算師從一本新書中獲得了同產業別的實際總損失數據。有五個具代表性風險的損失經驗如下：

	實際總損失
1	275,000
2	300,000
3	500,000
4	700,000
5	800,000

精算師建議結合上述兩個資訊，認為合併後的數據可以更準確地計算被保險人的回溯保險費(retrospective premium)。請評估精算師的陳述是否合理。

【參考解答】

(a)

Assume T = 1

B = e − (c − 1)E[A] + cI

E[A] = avg of 10 risk losses = 160,000

I =((280,000 − 240,000)+(440,000 − 240,000))/10 − (40,000 – 20,000)/10= 22,000

B = 10,000 − (1.05 − 1)(160,000) + (1.05)(22,000) = 25,100

G = 25,100 + 1.05 × 240,000 = $277,100

(b)

新數據顯示這五個風險擁有比最初的 10 個風險更大的損失，若合併它們，將會顯著改變我們定價風險的總損失分佈和預期損失。如果被保險人的規模與原來的 10 個風險相似(E [A] = $ 160k)，那麼包涵新數據將高估被保險人的預期損失和 insurance charge。

【題目出處】

Fisher “Individual Risk Rating”

Q9.（3.5 分）

被保險人正在考慮一個已發生賠款回溯費率(retrospectively rated)計畫和已付賠款的自負額計畫(deductible plan)，給定以下資訊：

已發生損失：

每次事故限額	$150,000
最高可計價損失	$500,000

已付賠款的自負額：

每次事故自負額	$150,000
累計自負額	$500,000

The following apply to both plans:

預計未受限制最終損失	$440,000
每事故限額之預期最終損失	$350,000
受每事故限額及總限額之預期最終損失	$325,000

以下適用於這兩個計畫：

固定承保費用(包括佣金和利潤)	$100,000
理賠費用佔損失百分比	10.0%
稅收乘數	1.05

(a)(1.5 分)

被保險人的角度來看，比較兩個計畫的預期保險總成本，請同時考慮保費和自負額。

(b)(1 分)

從被保險人的角度，請推薦上述兩個計畫中的一個，並提出簡要說明。

(c)(1 分)

請列舉兩項改變，會令(b)小題中推薦的計畫對被保險人更加有利，並提出簡要說明。

【參考解答】

(a)

Expected Retro prem 預期回溯保費= (440k × 1.1 + 100k) × 1.05 = $613,200

Expected Ded Policy losses 自負額保單預期損失

= 440k - 325k = 115k

Expected Ded Policy LAE 自負額保單預期理賠費用

= 10% × 440k = 44k

Expected Ded Policy Prem 自負額保單預期保費

= (115k + 44k + 100k) × 1.05= $271,950

Retro total cost of insurance 回溯保費計畫的成本= $613,200

Ded Plan total cost of insurance 自負額保單計畫的成本

= $271,950 prem + $325,000 retained = $596,950

The lower total cost for the deductible plan is due to the tax multiplier applying to a smaller premium based on just excess losses.

自負額計畫的總成本較低是由於稅收乘數適用與一個較小的保費基礎(因為保費僅來自於超額賠款)

(b)

Solution 1: Recommend the Deductible Plan

I would recommend the deductible plan as it results in lower expected costs for the insured, and a lower initial outgoing premium cash flow for the insured.

方案 1：推薦自負額計畫

我會建議自負額計畫，因為它使被保險人的預期成本降低，以及被保險人的初始流通在外保費(outgoing premium)現金流量較低。

Solution 2: Recommend the Retro Plan

I would recommend the retro plan as the insured can potentially have lower costs if they have very good experience, and the insured will be able to tax deduct the higher insurance initial premium.

方案 2：推薦回溯計畫(Retro Plan)

我會建議回溯計畫(Retro Plan)，因為損失經驗非常好，被保險人可能會有較低的成本，並且更高的保險初始保費可使被保險人享有更高的稅負扣除額。

(c)

Corresponding with the deductible plan:

Any 2 of:

• Increasing deductibles would lower premium, and would further reduce the cost of insurance since the tax multiplier would apply to a smaller premium.

• Changing to a self-insured retention with an excess policy would give the insured greater control over losses below the retention and would further reduce expected costs.

• Lowering deductibles would provide the insured more certainty in its total costs (though they would likely be higher as premium would increase).

(c)

與自負額計畫相對應：

任選兩個：

•增加自負額會降低保費，並且會進一步降低保險成本，因為稅收乘數適用於較低的保費。

•改為自我保險自負額(self-insured retention)搭配超額保單(excess policy)將使被保險人能夠更好地控制低於自留額的損失，並進一步降低預期成本。

•降低自負額將使被保險人的總成本更加確定(儘管隨著保費的增加，總成本可能會更高)。

Corresponding with the retro plan:

Any 2 of:

• Changing the retro to use the retrospective development option would allow for more predictable cash flows for the insured.

• Adding holdbacks to the retro plan would defer retrospective premium adjustments to a later date when losses have reached more maturity and would result in more predictable cash flows for the insured.

• Adding a minimum ratable loss could reduce the basic premium

與回溯計畫(retro plan)相對應：

任何兩個：

•更改回溯計畫，使其採用回溯發展選項(retrospective development option)將為被保險人提供更可預測的現金流。

•對追溯計畫加上阻止(holdback)條件，將追溯保費調整時間延後，使損失達到更接近發展成熟的日期，並為被保險人帶來更可預測的現金流量。

•增加最低比例的損失(minimum ratable loss)，可降低基本保費。

Q10.(3 分)

以下述超越機率曲線適用於保險公司的業務組合：

回歸期	發生超越機率曲線	損失
10,000	0.0001	100,000,000
500	0.0020	25,000,000
200	0.0050	10,000,000
100	0.0100	6,000,000
50	0.0200	3,500,000
33	0.0300	1,750,000
25	0.0400	750,000
20	0.0500	250,000

(a)(1 分)

保險公司可接受的風險等級為回歸期 250 年的最大可能損失，請計算回歸期 250 年的最大可能損失。

(b)(2 分)

保險公司決定依據以下條約購買財產巨災再保險合約，來保障為回歸期 500 年的最大可能損失：

採用比例性再保合約額，分出 30%至 20 百萬元的損失限額，再以非比例性合約分出：

- 第一層：2 百萬元以上的 3 百萬(excess of loss treaty $3 million xs $2million)，分出 100%
- 第二層：5 百萬元以上的 5 百萬(excess of loss treaty $5million xs $5million)，分出 90%
- 第三層：10 百萬元以上的 15 百萬(excess of loss treaty $15 million xs $10 million)，分出 75%

在合約期間，保險公司遭受了 22.5 百萬元的地震損失。

請計算分配給每個再保險合約的損失金額和簽單公司的淨保留損失。

【參考解答】

(a)

PML stands for Probable Maximum Loss, which is the largest loss likely to occur in a given period of time with a given probability of occurrence. The 1-in-250 year PML is the largest loss event expected to occur over a 250 year period. 1 / 250 = 0.004

PML 代表可能的最大損失，這是在給定的發生概率和一段時間內，所可能發生的最大損失，回歸期 250 年的最大可能損失是預計在 250 年內發生的最大損失

事件。1/250 = 0.004

1-in-250 year PML(可能的最大損失) = Loss(損失) at OEP of 0.004 = $10M + ($25M - $10M) ×(0.004−0.005)/(0.002−0.005) = $15M

(b)

Ceded to Quota Share = 30% × min($22.5M, $20M) = $6M

Remaining loss = $22.5M - $6M = $16.5M

分出給比例性再保合約之損失(Ceded to Quota Share)= 30%×Min(22.5 百萬元, 20 百萬元)= 6 百萬元

剩餘損失= 22.5 百萬元 - 6 百萬元= 16.5 百萬元

Layer	Insurer	Layer 1	Layer 2	Layer 3	Total
0-$2M	$2M				$2M
$2M-5M		(100%)$3M=$3M			$3M
$5M-10M	(10%)($5M)=$0.5M		(90%)($5M)=$4.5M		$5M
$10M-16.5M	(25%)($6.5M)=$1.625M			(75%)($6.5M)=$4.875M	$6.5M
Total	$4.125M	$3M	$4.5M	$4.875M	$16.5M

So the insurer retains $4.125M, $6M is ceded to the quota share, $3M is ceded to the first layer,$4.5M is ceded to the second layer, and $4.875M is ceded to the third layer.

因此，保險公司自留了 4.125 百萬元，6 百萬元元被分配給比例性再保合約的再保人，3 百萬元被分配給第一層的再保人，4.5 萬元被分配給第二層的再保人，4.875 百萬元被分配給第三層的再保人。

【題目出處】

Grossi “Catastrophe Modeling: A New Approach to Managing Risk”

Q11. (2 分)

一家再保險公司正在評估是否與一家簽單公司(primary insurer)簽訂 1 億元以上的 1 億萬元($100 million excess of $100 million)的非比例性巨災再保險合約。該再保險公司目前持有 17 億元的資金(capital)，並且需要持有足夠的資金才能在 250 年回歸期的事件(1-in-250 event)中倖存下來。如果沒有新的合約，再保險公司的可能性最大可能損失(PML)為 16.5 億元，且僅受颶風事故的影響。

給定以下內容：

•簽單公司的 PML 僅受颶風和地震危險的影響。

•簽單公司的年度總 PML(依回歸期區分)如下：

回歸期(年)	PML($000,000)
1,000	250
500	210
200	190
100	140
50	100
25	60
20	50
10	40
5	30

•簽單公司面臨最大的颶風事件損失是 90,000,000 元。

(a)(1 分)

計算簽單公司 250 年回歸期的事件的分出與淨自留損失。

(b)(1 分)

請評估再保險人是否應參與該合約。

【參考解答】

(a)

Assume PMLs are occurrence-based.

假設 PML 是基於事件的

Interpolate gross 250 year PML =190(210-190)/(1/200−1/500) × (1/200 − 1/250) = 196.67M

Ceded 250 year PML (loss in 100M xs 100M layer) = $96.67M

Net 250 year PML (gross - ceded) = $100M

內差出 250 年回歸期 PML =190(210-190)/(1/200−1/500) × (1/200 − 1/250) = 196.67M

分出損失 Ceded 250 year PML (loss in 100M xs 100M layer) = $96.67M

淨自留損失 Net 250 year PML (gross - ceded) = $100M

(b)

簽單公司原本面臨的最大的颶風事件，損失金額是 90 百萬元，不會達到再保險條約自留額 1 億元。 因此，此再保險條約只會暴露於地震風險。 而且由於目前再保險公司的 PML 16.5 億元完全由颶風驅動，任何地震都不會使此 PML 更高，因地震與颱風為獨立事件。 因此，再保險公司在這裡不存在與 250 年回歸期有關的資本問題，再保險公司應參與該合約。

【題目出處】

Grossi "Catastrophe Modeling: A New Approach to Managing Risk"

Q12.（2 分）

某再保險人在非比例合約(non-proportional treaty)下使用以下暴露曲線：

$$G(x) = \frac{\ln(0.2+0.02^{x}) - \ln(2.2)}{\ln(0.22) - \ln(2.2)}$$

根據該條約，該分保的最高自留額為 5000 萬元，最大可能的第一元(maximum possible first-dollar loss)損失為 1 億元。

A function with the form of

$$G(x) = \frac{\ln(a+b^{x}) - \ln(1+a)}{\ln(a+b) - \ln(a+1)}$$

has a derivative of

此函數的一階微分為

$$G'(x) = \frac{\frac{\ln(b)b^{x}}{a+b^{x}}}{\ln(a+b) - \ln(a+1)}$$

(a)(1 分)

請計算分保公司(cedant)所自留的純風險保費的比率。

(b)(1 分)

計算全損(total loss)的機率。

【題目有誤，本題送分】

Q13.（2 分）

保險公司的總損失經驗為以下分配函數：

$F(x) = x^{0.25}$ where $0 \le x \le 1$

(a) (1 分)

請從上述累積分佈函數(cumulative distribution function.)導出曝露曲線(exposure curve)。

(b) (1 分)

給定最大可能損失為 4,000,000 元，使用上述(a)部分中的資訊計算在 1,000,000 元以上的 1,000,000 元那層(the layer $1,000,000 excess of $1,000,000)的純風險保費比率。

【參考解答】

(a)

$$G(d) = \frac{\int_0^d [1-F(x)]dx}{\int_0^1 [1-F(x)]dx} = \frac{\int_0^d [1-x^{0.25}]dx}{\int_0^1 [1-x^{0.25}]dx} = \frac{\left[x-\left(\frac{1}{1.25}\right)x^{1.25}\right]\Big|_0^d}{\left[x-\left(\frac{1}{1.25}\right)x^{1.25}\right]\Big|_0^1} = \frac{d-0.8d^{1.25}}{1-(0.8)(1)} = 5d-4d^{1.25}$$

(b)

Retention 自留 / Maximum Possible Loss 最大可能損失

= $1,000,000 / $4,000,000 = 0.25

Exposure Factor 暴露因子 = $G(0.25) = (5)(0.25) - (4)(0.25)^{1.25} = 0.543$

(Retention 自留 + Limit 限額) / Maximum Possible Loss 最大可能損失

= ($1,000,000 + $1,000,000) / $4,000,000 = 0.5

Exposure Factor 暴露因子 = $G(0.5) = (5)(0.5) - (4)(0.5)^{1.25} = 0.818$

Exposure Factor for layer 該層的曝露因子= 0.818- 0.543 = 0.275

【題目出處】

Bernegger “The Swiss Re Exposure Curves and the MBBEFD Distribution Class”

Q14.（4 分）

再保險公司的精算師使用以下暴露曲線(exposure curve)為非比例性合約定價，假設 b = 0.1：

$G(x) = \frac{1-b^x}{1-b}, 0 \le x \le 1$

再保險公司的最大可能損失為 5000 萬元，而且保險人保留的純風險保險為 55%。

計算此合約的最高自留額。

【參考解答】

我假設最大可能的基礎損失(ground-up loss)等於保留自留額加上再保險公司 MPL。

Cedant's share = G(retention / MPL) - G(0) = 55%

G(0) = 0

$G(retention/MPL) = \frac{1-0.1^{retention/MPL}}{1-0.1} = 55\%$

Retention / MPL = 0.296709

再保險公司將支付超過自留額的 5000 萬元，一直到 MPL(根據我們上面的假設)。

因此，再保險公司承保的 5000 萬元將佔 MPL 的 1 - 0.2697 = 70.3%。

MPL = $50M / 70.3% = $71.09M

因此，分出公司的最大自留額為 7109 萬元--5000 萬元=2109 萬元。

【題目出處】

Bernegger "The Swiss Re Exposure Curves and the MBBEFD Distribution Class"

Q15. (3 分)

某組具有同質性之風險表現出以下風險特徵：

曝露分佈	損失佔最大可能損失(MPL)的百分比
80%	0%
6%	25%
8%	50%
4%	75%
2%	100%

(a) (1 分)

繪製曝露曲線 G(x)，包括軸與曲線上的點。

(b) (1 分)

使用以下信息，計算出適合此曝露曲線的 MBBEFD 分佈參數 b 和 g：

Calculated value of parameter *b*									
Parameter	*μ*								
g	40.0%	42.5%	45.0%	47.5%	50.0%	52.5%	55.0%	57.5%	60.0%
50.0	0.0023	0.0017	0.0013	0.0009	0.0005	0.0003	0.0002	0.0001	0.0001
25.0	0.0096	0.0073	0.0055	0.0041	0.0022	0.0015	0.0010	0.0006	0.0004
16.7	0.0239	0.0181	0.0137	0.0103	0.0057	0.0039	0.0026	0.0017	0.0010
12.5	0.0483	0.0365	0.0276	0.0209	0.0118	0.0081	0.0055	0.0036	0.0023
10.0	0.0877	0.0659	0.0497	0.0375	0.0212	0.0147	0.0100	0.0067	0.0044
8.3	0.1498	0.1116	0.0836	0.0628	0.0354	0.0245	0.0168	0.0113	0.0075
7.1	0.2470	0.1817	0.1347	0.1004	0.0561	0.0389	0.0267	0.0181	0.0121
6.3	0.4000	0.2892	0.2116	0.1561	0.0861	0.0594	0.0408	0.0277	0.0185
5.6	0.6446	0.4557	0.3277	0.2385	0.1291	0.0885	0.0605	0.0411	0.0276
5.0	1.0469	0.7190	0.5055	0.3616	0.1910	0.1297	0.0882	0.0597	0.0400

(c) (1 分)

再保險公司使用此曝露曲線對 1 億元限額(underlying limit)的再保險合約做定價，請計算 2000 萬元以上 8000 萬元(the layer $80 million excess of $20 million)損失的比例。

【參考解答】

(a)

E[X;0%] = 0

E[X;25%] = (80%)(0%) + (6% + 8% + 4% + 2%)(25%) = 0.05

E[X;50%] = (80%)(0%) + (6%)(25%) + (8% + 4% + 2%)(50%) = 0.085

E[X;75%] = (80%)(0%) + (6%)(25%) + (8%)(50%) + (4% + 2%)(75%) = 0.1

E[X] = (80%)(0%) + (6%)(25%) + (8%)(50%) + (4%)(75%) + (2%)(100%) = 0.105

G(x) = E[X;x] / E[X] = E[X;x] / 0.105

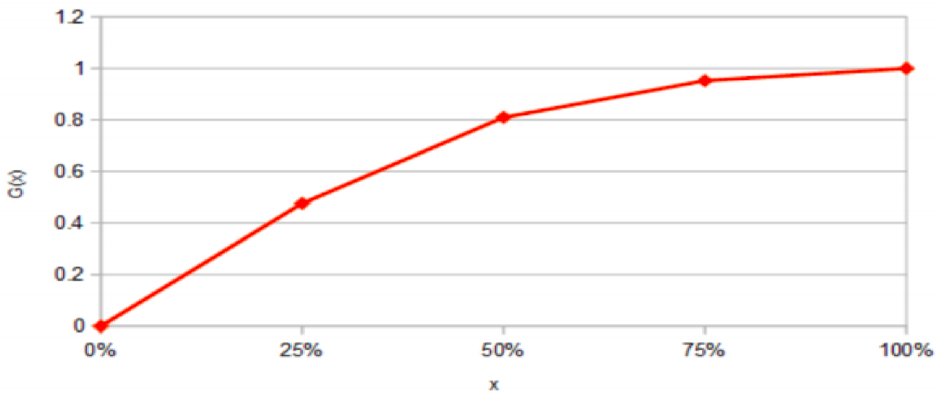
G(0) = 0

G(0.25) = 0.476

G(0.5) = 0.810

G(0.75) = 0.952

G(1) = 1



(b)

我假設此分配可用一個簡單的方法轉換成損失幅度分配，方法為：刪除損失 0 的情形並重新計算機率。

p = total loss prob given a non-zero claim occurs

發生非零索賠時的全損(total loss)機率= 2% / (1 - 80%) = 10%

g = 1/p = 10

μ = E[X|X > 0] = 0.105 / (1 - 80%) = 52.5%

查找 g 和 μ 得到 b = 0.0147

(c)

$40M / $100M = 20%

($40M + $60M) / $100M =100%

Portion of loss in layer層中損失的部分= G(100%) - G(20%) = 1- 0.476*.8 = 0.619

【題目出處】

Bernegger “The Swiss Re Exposure Curves and the MBBEFD Distribution Class”

Q16.（2 分）

基本保費在如下條件如何作為保證成本保費(guaranteed-cost premium)的百分比變化？

(a) 損失轉換因子上升(The loss conversion factor increases)？(0.5 分)

(b) 損失限額增加？(0.5 分)

(c) 最高保費或最高可計價損失(maximum ratable loss)增加？(0.5 分)

(d) 最低保費或最低可計價(minimum ratable loss)損失增加？(0.5 分)

【參考解答】

(a) 隨著損失轉換因子的增加，費用從基本保費轉移到損失，故基本保費減少。

(b) 隨著損失限額的增加，每次發生的超額風險的費用減少，基本保費減少。

(c) 隨著最高保費或最大可計價(ratable)損失金額的增加，總超額風險的費用減少，基本保費減少。

(d) 隨著最低保費或最低可計價(ratable)損失金額的增加，總超額風險淨保費(即保險費淨額)減少，基本保費減少。

【題目出處】

Fisher “Individual Risk Rating”

Q17. （1 分）

損失敏感計劃(loss-sensitive dividend plan)為什麼會不平衡？

【參考解答】

損失敏感計劃不平衡的原因是如果損失經驗好於預期，被保險人將獲得股息，但如果損失經驗比預期差，則被保險人不會產生任何額外費用。

【題目出處】

Fisher “Individual Risk Rating”

Q18.（1 分）

在什麼條件下，回溯費率計劃(retrospective rating plan)和高自負額計劃(large deductible plan)的風險轉移是一樣的？

【參考解答】

當在以下的狀況，回溯費率計劃和高自負額計劃的風險轉移是相同的

- 回溯費率計劃的損失限額等於每次事件的自負額
- 回溯費率計劃的最高可計價損失(maximum ratable loss)金額等於高自負額計劃的總自負限額(aggregate deductible limit)。
- 回溯費率沒有最低可計價損失(minimum ratable loss)金額。

【題目出處】

Fisher “Individual Risk Rating”

Q19.（2 分）

使用巨災模型來決定費率時，存在哪四種開放議題(Open Issues)？(2 分)

【參考解答】

Regulatory Acceptance
Public Acceptance
Actuarial Acceptance
Model-to-Model Variance

【題目出處】

Grossi, P. and Kunreuther, H., Editors, Catastrophe Modeling: A New Approach to Managing Risk, 2005, Springer, Chapter 5

Q20.（5 分）

巨災風險組合管理(Portfolio Management)：

a. 在處理巨災風險時，組合管理者(Portfolio Manager)會面臨哪二個重要問題？(2 分)

b. 上述二個重要問題，要如何處理？(1 分)

c. 巨災風險組合管理可分為微觀管理(Micromanagement)和宏觀管理(Macro management)，兩者有何不同？(2 分)

【參考解答】

a. What is the average annual loss (AAL) and what is the likelihood that the company may become insolvent?

b. To address both of these issues, it is critical to adequately model the right hand tail of the EP curve where the loss is large and there is a significant amount of uncertainty.

c. Micromanagement addresses individual policies or even locations, while macro management considers the aggregate portfolio.

【題目出處】

Grossi, P. and Kunreuther, H., Editors, Catastrophe Modeling: A New Approach to Managing Risk, 2005, Springer, Chapter 6

Q21.（5 分）

一間保險公司到觀察某險種的總額損失經驗(Ground-up claim experience)如下：

賠案件數(Claim counts)(N)服從 Poisson 分配，其中$\lambda = 4{,}000$。

賠案金額(Claim size)(X)服從 Pareto 分配，其中$\alpha = 2$，$\beta = 6{,}000$。

這間保險公司會支付損失超過 1 萬元的部分(excess of $10,000)。

給定下列公式：

Pareto 分配：

$$F(x) = 1 - \left(\frac{\beta}{x+\beta}\right)^{\alpha} \qquad \mathrm{E}[X;x] = \frac{\beta}{\alpha-1}\left[1 - \left(\frac{\beta}{x+\beta}\right)^{\alpha-1}\right]$$

a. 計算損失超過 1 萬元的預期賠案件數(expected number of claims in excess of $10,000)。(1 分)

b. 假設損失金額服從均勻每年通膨率 3%(uniform annual inflation rate of 3%)。計算總額損失件數改變率(the rate at which ground-up claim counts must change)，使得預期每年總超額損失不變(no change in expected annual total aggregate excess losses)。(4 分)

【參考解答】

a. $\mathrm{F}(10{,}000) = 1 - \left(\frac{6{,}000}{10{,}000+6{,}000}\right)^2 = 0.859 \qquad 4{,}000 \times (1 - 0.859) = \mathbf{564}$

b. $\mathrm{E}[\mathrm{X}] = \frac{6{,}000}{2-1} = 6{,}000 \qquad \mathrm{E}[\mathrm{X}; 10{,}000] = \frac{6{,}000}{2-1} \times \left(1 - \left(\frac{6{,}000}{10{,}000+6{,}000}\right)^{2-1}\right) = 3{,}750$

$$\mathrm{E}[\mathrm{X}; 10{,}000/1.03] = \frac{6{,}000}{2-1} \times \left(1 - \left(\frac{6{,}000}{10{,}000/1.03 + 6{,}000}\right)^{2-1}\right) = 3{,}708$$

$$\frac{1.03 \times (E[X] - \mathrm{E}[\mathrm{X}; 10{,}000/1.03])}{E[X] - \mathrm{E}[\mathrm{X}; 10{,}000]} = \frac{1.03 \times (6{,}000 - 3{,}708)}{6{,}000 - 3{,}750} = 1.049$$

$$\frac{1}{1.049} - 1 = \mathbf{-4.67\%}$$

【題目出處】

LEARNING OBJECTIVE(S): B1, B2

Q22. (5 分)

一位精算師使用風險調整後增加限額因子(risk-adjusted limit factors)來做為定價的風險考量。他的同事建議另一個方法，使用非風險調整後增加限額因子(non-risk-adjusted limit factors)，並且隨著保額(policy limit)調整利潤和意外加載(profit and contingency load)。

在下面各方面，比較這二個方法的差異：

- 正確性(Accuracy)。(2 分)
- 計算容易(Ease of calculation)。(1.5 分)
- 透明度(Clarity)。(1.5 分)

【參考解答】

Sample 1

Accuracy – the risk adjusted ILF is more accurate than the varying profit and contingency because it is more explicitly calculating the risk load as limits increase. The profit varying is more arbitrary.

Ease of Calculation – The risk load is much more computationally difficult. The profit can be more judgmentally selected.

Clarity – The risk load has a foundation in mathematics so would be more clear to a trained eye. But a lay person would likely better understand the profit variation.

【題目出處】

LEARNING OBJECTIVE(S): B1

Q23.（5 分）

有一間保險公司使用二種不同的方法，來對超額層保險合約(excess layer insurance contract)定價：

- 建立經驗表 M(Empirical construction of Table M)。
- 連續近似模型近似總損失分配(Approximating the distribution of aggregate losses with a continuous approximation model)。

a. 簡述連續近似模型(continuous approximation model)的二個潛在缺點。(2 分)

b. 完整說明建立經驗表 M 的過程，以及如何用這個表來估計超額層的整體損失。(3 分)

【參考解答】

a. Sample 1

1) Lack of data. The distribution estimated can be difficult or far away from reality due to sparse data.

2) Need more calculation. Maybe time consuming.

b. Sample 1

1) For all n risks (assume similar sizes), calculate the average aggregate loss amount.

2) Order the risk in order of their entry ratio actual/expected.

3) For each desired entry ratio (ri), determine the % of risks whose entry ratio is above that ri.

4) Starting from the highest ratio, assume its charge is 0. The charge for rn−1 can be determined using the layer method referred in the formula: charge (rn)=charge (rn+1) + (rn+1 -rn)(% of risks above rn). Repeat until all charges are complete. If desired, the table M savings can be included by computing saving (rn) = charge (rn)+ rn -1. The aggregate excess loss cost of an layer at G can be determined by finding the entry ratio rG and multiplying the charge (rG)* E(A)

【題目出處】

LEARNING OBJECTIVE(S): B2

Q24.（5 分）

一張保單有定額式自負額 M(flat dollar deductible M)，以及一次損失保險人最大賠付金額 N(maximum payout on a loss by the insurer of N)。

a. 畫出李氏圖(Lee diagram)來表示這張保單的預期已發生損失金額。標示下列各項：(1 分)

i. 軸(The axes)

ii 自負額(The deductible amount)

iii 保額(The policy limit)

iv. 期望被保險損失(The expected insured loss)

b.假設累積損失服從分配 F(x)。寫出這張保單的包含損失(covered losses)公式，分別使用：(1 分)

i. 分層法(The layer method)

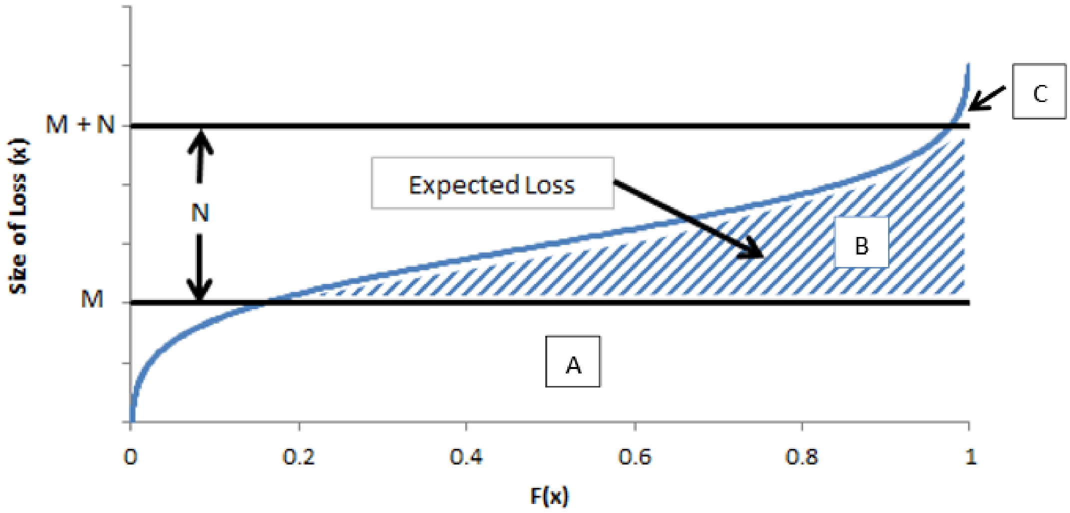
ii 尺寸法(The size method)

c. 簡要說明何時分層法較適合？何時尺寸法較適合？(1 分)

d. 使用李氏圖來表示增加限額因子的一致性檢定(the consistency test of ILFs)。(2 分)

【參考解答】

a.



Expected Insured Loss = B

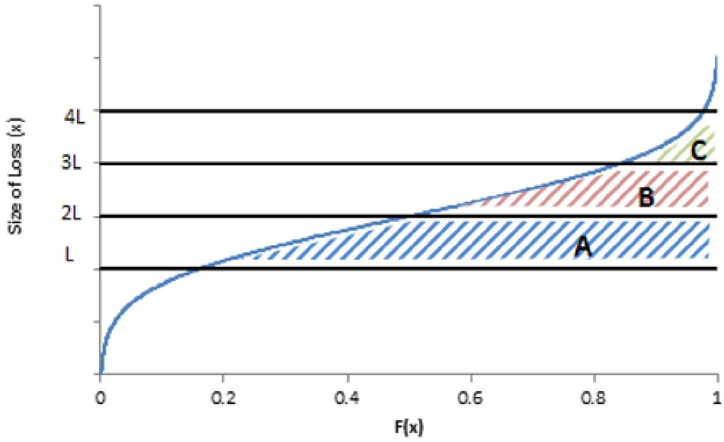
b.

i. $\int_{M}^{M+N} 1 - F(x)dx$

ii. $\int_{M}^{M+N} xdF(x) + (M+N) * \big(1 - F(M+N)\big) - M * \big(1 - F(M)\big)$

c. The layer method may be preferred when the survival function is easy to integrate. The size method may be preferred when empirical data is not available and integrals need to be evaluated algebraically.

d.



The consistency test of ILFs states that the premium calculated from the layer formula applied to successive excess layers of constant width is a decreasing function of the attachment point limit. Area A > Area B > Area C therefore Premium for Layer L to 2L > Premium for Layer 2L to 3L > Premium for Layer 3L to 4L and the consistency test is satisfied.

【題目出處】

LEARNING OBJECTIVE(S): B1b, B2b

Q25.（2.5 分）

給定下列資訊：

- 可分配理賠費用是賠付金額(indemnity amount)的 20%。
- 變異數法(The variance method)已被選定，包含對增加限額因子的風險加成(risk load in the Increased Limits Factors (ILFs))，而且 k = 0.000064，δ = 0。

Limit, l	E[X; l]	E[X^2; l]
1,000	840	790,123
5,000	2,485	9,467,456

a. 計算下列內容：(2 分)

- 對每一限額的風險加成(The risk loads for each limit)
- 包含及不包含風險加成的限額 5,000 增加限額因子(The ILF with and without risk load for the 5,000 limit)

b. 假設群體(portfolio)中有 75%是限額 1,000，25%是限額 5,000，使用包含風險加成的增加限額因子(而不是不包含風險加成的增加限額因子)來決定對整體保費的影響。(determine the overall impact on premium by using the ILFs with the risk loads instead of the ILFs without risk load)（0.5 分）

【參考解答】

a. Risk load $= \mathrm{k}\mathrm{E}[X^2; l] + \delta[\mathrm{E}[X; l]^2]$

$\delta = 0$

risk load @ 1000 = 0.000064[790,123] = **50.568**

risk load @ 5000 = 0.000064[9,467,456] = **605.917**

$$\text{ILF(5k) w/o risk load} = \frac{2{,}485 \times 1.20}{840 \times 1.20} = \mathbf{2.958}$$

$$\text{ILF(5k) with risk load} = \frac{2{,}485 \times 1.20 + 605.917}{840 \times 1.20 + 50.568} = \mathbf{3.389}$$

b. $\frac{0.75X + 0.25(3.389X)}{0.75X + 0.25(2.958X)} - 1 = +\mathbf{7.23}\%$

【題目出處】

LEARNING OBJECTIVE(S): B1b

Q26.（2.5 分）

依保險人銷售起賠額(attachment point)為 5,000，層限額(layer limit)為 5,000 的保障(coverage)。下列的表呈現在這年的不同限額下，預期累積幅度分配(expected cumulative severity distribution)和限額預期幅度(limited expected severity)的結果：

l	F(l)	E[X; l]
4,167	0.842	1,807
5,000	0.859	1,875
6,000	0.875	1,941
8,333	0.938	2,256
10,000	0.947	2,308
12,000	0.954	2,357

精算師預期對所有賠案(all claim size)，明年損失幅度增加 20%。

a. 計算在這層中的賠案頻率變動百分比(the percentage change in frequency of claims in the layer)。（1 分）

b. 計算這層中的純保費變動百分比(the percentage change in pure premiums in the layer)。（1.5 分）

【參考解答】

a. Sample 1

Current Layer Frequency = F(10,000)–F(5,000) = 0.947 – 0.958 = 0.088

Future Layer Frequency = F(10,000/1.2) – F(5,000/1.2) = 0.938 – 0.842 = 0.096

% Change Frequency = **9.09%**

Sample 2

$$t_n = \frac{S\left(\frac{a}{t}\right)}{S(a)} = \frac{S(4{,}167)}{S(5{,}000)} = \frac{1-.842}{1-.859} = 1.121 \rightarrow \mathbf{12.1\%}$$

b.

$$t_s = \frac{t\left(E\left[X;\frac{10{,}000}{1.2}\right] - E\left[X;\frac{5{,}000}{1.2}\right]\right)}{E[X;10{,}000] - E[X;5{,}000]} - 1 = \frac{1.2(2{,}256-1{,}807)}{2{,}308-1{,}875} - 1 = \mathbf{24.43\%}$$

【題目出處】

LEARNING OBJECTIVE(S): B1

Q27.（5 分）

一再保人從一大保險公司取得下列資訊：

賠案大小範圍	預期賠案件數	預期最終損失(千元)
$0 to $1,000,000	19,000	$6,750,500
$1,000,001 to $2,000,000	359	$525,300
$2,000,001 to $3,000,000	230	$566,500
$3,000,001 to $4,000,000	147	$507,700
$4,000,001 以上	264	$1,650,000
合計	20,000	$10,000,000

這再保人和此保險公司簽訂超額賠款契約(excess of loss contract)。再保人會賠付損失超過每一賠案自付額(per claim retention)$5,000,000 的部分。

a. 畫出損失$1,000,000、$2,000,000、$3,000,000 和$4,000,000 的超額幅度函數(excess severity function)圖形。（4 分）

b. 計算在此預定契約(proposed contract)下的再保人預期損失(reinsurer's expected losses)。（1 分）

【參考解答】

a.

Limit, L	1-F(L)	E[X;L]	XS Severity at L
1,000,000	0.05	387,525	2,249,500
2,000,000	0.3205	427,890	2,249,922
3,000,000	0.2055	453,765	2,249,878
4,000,000	0.0132	470,300	2,250,000

E[X] = 10,000,000,000/20,000 = 500,000

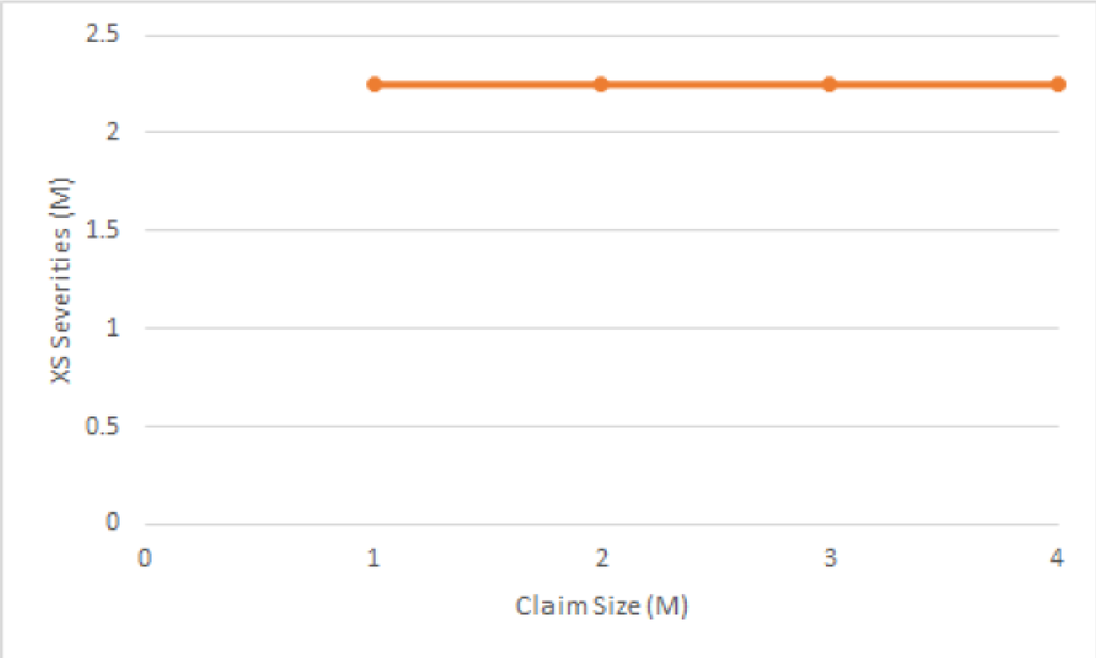
E[X; 1M] = (6,750,500,000 + 1000(1,000,000))/20,000 = 387,525

E[X; 2M] = (6,750,500,000 + 525,300,000 + 2,000,000(230 + 147 + 264))/20,000 = 427,890

E[X; 3M] = (6,750,500,000 + 525,300,000 + 566,500,000 + 3,000,000(147 + 264))/20,000 = 453,765

E[X; 4M] = (6750,500,000 + 525,300,000 + 566,500,000 + 507,700,000 + 264(4,000,000))/20,000 =470,300

XS Severity at L = (E[X] – E[X; L])/(1-F(L))



b. From a. excess severity is flat

→ Losses follow exponential distribution above 1M with beta = 2.25M

→ e(5M) = e(4M) = … = e(1M) = 2.25M

→ P(X>5M|X>1M) = $e^{-(5-1)M/2.25M}$ = 0.169

→ E[N|X>5M] = (20k – 19k) * 0.169 = 169.01

→ Reinsurer E[L] = 169.01 * 2.25M = **380.28M**

【題目出處】

LEARNING OBJECTIVE(S): B1d